Building the City: Urban Transition and Institutional Frictions

J. Vernon Henderson (LSE and SERC)
Tanner Regan (LSE and SERC)
Anthony J. Venables (Oxford and SERC)

Revised April 2017
(Replaced April 2016 version)
This work is part of the research programme of the Urban Research Programme of the Centre for Economic Performance funded by a grant from the Economic and Social Research Council (ESRC). The views expressed are those of the authors and do not represent the views of the ESRC.

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J. Vernon Henderson*
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Anthony J. Venables***

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* London School of Economics and Spatial Economic Research Centre, LSE
** London School of Economics and Spatial Economic Research Centre, LSE
*** University of Oxford and Spatial Economic Research Centre, LSE

We gratefully acknowledge the support of an Africa Research Program on Spatial Development of Cities at LSE and Oxford funded by the Multi Donor Trust Fund on Sustainable Urbanization of the World Bank and supported by the UK Department for International Development. We acknowledge the excellent research assistance of Ilia Samsonov and Piero Montebruno. Thanks to seminar participants at LSE, Lausanne, Oxford, Helsinki and the NBER.
Abstract

We model the evolution of the built environment over time and space in a growing city. Durable formal sector buildings can be built high, unlike informal that are malleable. With city growth, areas are initially developed informally, then formally, and then redeveloped periodically. Political/institutional costs of converting slum to formal sector usage delay development. We analyse Nairobi for 2003/4 and 2015 using unique data on building footprints and heights. In that period, volume in the city core increases by 50-60% driven by increased height of redeveloped buildings. We calibrate the high welfare cost of delayed conversion of older slums.

Keywords: city, urban, urban growth, slum development, urban structure, urban form, housing investment, capital durability
JEL Classifications: O14; O18; R1; R3
1. Introduction

This paper examines building development and redevelopment in a growing city, and the welfare costs of ‘artificial’ land market frictions. We develop a model of a growing city which may have both informal (slum) and formal sectors, the latter containing durable buildings for which investment decisions are taken on the basis of expectations about the future growth of the city. We study urban transition as the city expands and land goes through phases of development and redevelopment, and we look at the welfare costs of inefficiencies in this transition. The model is applied to a unique data set on the built environment of Nairobi for 2003/4 and 2015, a fast growing metropolitan area with a core population of about 4 million and a greater urban area population of about 6.5 million. With the data we estimate key parameters to calibrate the model and hence calculate a measure of the welfare costs of institutional frictions in land markets which plague many cities in the developing world. In doing all this, we develop a novel set of facts about the dynamics of the built environment in a large developing country city.

We believe the model, methodology, and key empirical facts have general applicability to many cities in the developing world. We note that about 2/3 of the private capital stock of developing country nations are buildings (World Bank, 2006), which are disproportionately found in major cities. We know little about issues with the evolution of the urban built environment in developing country contexts, even though this is the major form of aggregate national investment.

Building on the standard monocentric model of a city (see Duranton and Puga, 2015 for a review) and the dynamic formulation by Braid (2001), our model captures the following features. First, the city is growing in income, population and area. Second, the city contains ‘formal’ or modern structures. Formal buildings involve sunk capital costs; can be built tall; and cannot be modified once constructed. As the city grows there will be periodic demolition and redevelopment of formal areas in response to rising land rents driven by growth fundamentals. Third, the city contains informal structures, which we sometimes refer to as slums. Given the technology and materials used in construction, slum buildings do not involve sunk capital; are not likely to be built tall; and their volume can be continuously adjusted over time. Finally, and most critically, there is a cost of conversion of informal to formal land use; this formalisation cost will vary across properties in the city. We will think of this formalisation cost as largely an artificial one, created by institutional and political frictions in land markets.

We show that, as the city grows, land will initially be developed with informal structures which are then replaced by formal structures, which will themselves be subject to intermittent redevelopment. The share of urban population in informal structures will generally decline through time. This decline is a consequence of rising land values (and hence a greater return
to achieving density by building upwards) as the city expands. However, formalisation costs mean that informal structures may be very persistent; and spatial heterogeneity of these costs mean that they will continue to exist alongside formal structures, having long-lasting implications for the fabric of the city.

For Nairobi we have a detailed data base of buildings. We know the footprints of all buildings throughout the urban area for 2003/4 and 2015 based on digital tracings from aerial photos. For 2015 we also know heights of these buildings based on LiDAR data, and we have high resolution satellite data for 2004 and 2013. The primary measure that we work with is the total volume of building space (height x built cover) per unit area, varying across the city and over time. We use the data based on aerial photos to analyse how Nairobi transforms from 2003/4 to 2015, tracing demolition, development and redevelopment at all locations. Based on mappings done for the city, we can divide the land into slum and formal sector land, so as to allocate buildings between the formal and informal sectors. In addition to this data on the built fabric of the city, we have data on land prices for 2015 and housing rents for 2012 by location which we use in our welfare calculations.

Nairobi conforms to predictions of our model and standard ones in the literature that, in the formal sector, house rents and land prices decline with distance to the centre and building heights and volume per unit area decrease with distance to the centre. Beyond that, we derive a novel set of predictions from the model and corresponding facts. We start with the cross-section. Consistent with the model, slums provide housing volume with high built coverage to area ratio and low height while, in contrast, the formal sector provides volume with height but comparatively low cover to area ratios. Comparing slum vs formal sector volume, in the core part of the city, slum and formal sectors provide a similar stock of built volume per unit area, albeit with slums at lower quality of building materials and amenities. In contrast to formal areas, the slum rent gradient is flat or even modestly rises with distance from the centre. Our model suggests that this is due to greater ‘crowding’ near the centre, as slum building volume is provided by increased cover.

For dynamics, the city changes dramatically from 2003/4 to 2015. There is rapid growth, with total built volume just within the 2003/4 city effective boundary growing at about 4% a year, a substantial rate of capital accumulation. And not surprisingly, there is a faster growth rate at the extensive margin, building beyond the 2003/4 effective boundary. At 1-6 km from the city centre, redevelopment of formal sector buildings into taller new buildings accounts for large volume increases, with the total net increase in volume from just redevelopment as a fraction of initial volume peaking at 35% at about 3 km out, with in-fill adding another 20%. Throughout the core of the city, there is substantial churning, with about 35% of formal sector buildings from 2003/4 being demolished. Towards the city edge there is enormous new development, or in-fill, to also help accommodate urban growth.
For the informal sector, there are no slums very near the city centre and an increasing proportion towards the city fringe, as predicted by the model. The rate of increase in slum sector building volume is just below that of the formal sector. While there is extensive churning and redevelopment within existing slums there is little mid-city development of slums into formal sector buildings. We explore the institutional context of Nairobi, which suggests that there are high formalisation costs in traditional mid-city slums and a significant amount of land is not in its best and highest value use. Using the calibrated model, we calculate the welfare cost of delaying conversion of mid-city slum plots to formal sector use. We find that, even if slumlords are paid the value of their land in perpetual slum use (although they have no legal claim to this), there would be an additional surplus of about $13,000 (2015 US $) per slum household in a context where they spend typically (median) about $260 a year on housing.

There are four novel aspects to the paper. First is the modelling. While Braid (2001) has a dynamic monocentric model with durable capital, no dynamic model deals with informality and formalisation costs. Second are the data. While there is work on the USA using demographic census data to analyse redevelopment over of periods of time (Brueckner and Rosenthal 2009), no work we know of utilizes city-wide data on individual buildings, with demolition, redevelopment, and infill used to model and detail the changes in the urban landscape. Third we develop a methodology to calculate the welfare cost of institutional frictions. This highlights the role of policy for fast growing cities with major market land failures that deter investment, because of lack of transparency and weak institutions governing land markets. Finally, we develop a new set of facts about city development and redevelopment of the built environment for a major developing country city.

The paper is organised as follows. The basic model and core theoretical results are set out in section 2. Section 3 presents data and the analysis of Nairobi, looking at the urban cross-section and patterns of change, and providing estimates of the parameters used in calibration. Section 4 develops the welfare measures used to characterize misallocation of land between older Nairobi slums and the formal sector. Section 5 discusses additional features of the city and Section 6 concludes.

2. Theory

In this section we present the model of a growing city, focusing on investment decisions and consequent patterns of land use and urban density. The analysis assumes that housing rent increases at an exogenous rate through time, and in the appendix we show how this can be endogenised in an open city equilibrium. Section 2.1 analyses building decisions associated with each of the slum and formal sector technologies. Section 2.2 focuses on a particular
point in the city and examines its evolution through time, as it transitions from agricultural use to informal development, then formalises and goes through successive waves of formal sector demolition and reconstruction. Section 2.3 shows how this path varies across points in the city, giving a complete description of both the cross-section of the city and its evolution through time. Section 2.4 adds some frictions to the model, looking at the role of expectations and focusing on how barriers to conversion from informal to formal development can lead to a ‘hotchpotch’: co-existence of different building types and sizes throughout the city. Section 2.5 sets out the cross-section predictions of the model, and shows how the empirics reveal key parameters of the theory model. Combining theory and empirics provides the methodology for measurement of the welfare cost of the inhibited development of slums near the centre of Nairobi, carried out in Section 4.

2.1 Building technology and housing supply

There are two distinct building technologies, formal and informal. Both can vary the amount of building volume they supply per unit land, but do so in different ways. The formal sector \((F)\) can build tall, and the informal sector \((I)\) can ‘crowd’, increasing cover, the proportion of land that is covered by building footprint. The volume of building delivered on a unit of land at a particular place, \(x\), and time \(t\), is the product of height and cover, \(v_i(x,t) = h_i(x,t)c_i(x,t)\), \(i = I, F\).

Informal sector construction materials are malleable and construction costs are a flow, occurring continuously through the life of the structure. This can be thought of as either the rental on ‘Lego blocks’ or ‘Meccano parts’ used in construction or as the cost of material whose life is one instant. The informal sector is unable to build tall so has height fixed at \(h_I = 1\). It can however increase the proportion of each unit of land that is covered with buildings, so \(v_I(x,t) = c_I(x,t)\). We assume that construction costs per unit volume are constant \(\kappa_i\), so construction costs per unit land are \(\kappa_i v_i(x,t)\). However, crowding has the effect of reducing the quality of housing. We capture this by supposing that the rent (and willingness to pay) for a unit of informal housing is the product of two elements; the rent of informal housing of unit quality at place \(x\) at date \(t\), \(p_i(x,t)\), and a quality or amenity factor, \(a(v_i(x,t))\), diminishing and convex in crowding (as measured by volume = cover per unit land). With this, land rent (i.e. housing rent minus construction cost times volume per unit land), is

\[
r_j(x,t) = [p_j(x,t)a(v_j(x,t)) - \kappa_j]v_j(x,t).
\]  

(1)
The volume of housing supplied is chosen to maximise land rent, taking \( p_j(x,t) \) as exogenous and internalising the effect of crowding on quality, \( a(v_j(x,t)) \). The first order condition is the equality of marginal revenue to marginal cost,

\[
\frac{\partial r_j(x,t)}{\partial v_j(x,t)} = p_j(x,t)a(v_j(x,t))\left[1 + v_j(x,t)a'(v_j(x,t))/a(v_j(x,t))\right] - \kappa_j = 0. \tag{2}
\]

If informal house quality is isoelastic in cover, \( a(v_j(x,t)) = a_0 v_j(x,t)^{(1-\alpha)/\alpha} \), \( \alpha > 1 \), then optimally chosen volume and maximised rent are respectively

\[
v_j(x,t) = \left[\frac{a_0 p_j(x,t)}{\kappa_j \alpha} \right]^{\frac{\alpha}{\alpha-1}}, \quad r_j(x,t) = \kappa_j \left(\alpha - 1\right) \left[\frac{a_0 p_j(x,t)}{\kappa_j \alpha}\right]^{\frac{\alpha}{\alpha-1}}. \tag{3}
\]

It follows that informal sector house rent adjusted for quality is constant throughout the city, \( p_j(x,t)a(v_j(x,t)) = \alpha \kappa_j \). Essentially, and as we will see later in the data, increased crowding near the centre will offset the advantage of improved access. Furthermore, construction costs and land rent are respectively share \( 1/\alpha \) and \((1-1/\alpha)\) of revenue earned by informal sector housing, so

\[
r_j(x,t) = \left[1 - 1/\alpha\right]p_j(x,t)a(v_j(x,t))v_j(x,t). \tag{4}
\]

The formal sector differs in a number of respects. First, buildings are ‘putty-clay’, malleable at the date of construction but not thereafter. We assume that formal sector land cover is not a choice variable but is set exogenously at \( c_F = 1 \), and that volume is achieved by choice of height, \( v_F(x,\tau_i) = h_F(x,\tau_i) \). This is chosen at date of construction, denoted \( \tau_i \), and then fixed for the life of the structure, i.e. until demolition at date \( \tau_{i+1} \), where subscript \( i = 1,2,... \) is used to denote successive redevelopments of formal structures. Construction costs per unit land are one-off and sunk, and are an increasing and convex function of building volume on that land, \( k\left(v(x,\tau_i)\right), k' > 0, k'' > 0 \). Demolition incurs neither costs nor benefits as materials cannot be recycled back to putty.

This sunk cost of construction differs fundamentally from the flow cost in the slum sector, and we think captures key differences in construction technology. In Nairobi, from the 2009 Census, formal and slum sector wall materials are distinctly different. In slums, the majority (about 55%) of housing walls are corrugated iron sheets which can be easily reconfigured like Meccano parts; most other slum housing involves mud construction (about 20%) and other material with short duration. Both sets of materials are not sufficiently load bearing to allow much in the way of height. In contrast, over 90% of formal sector housing is made of stone or some type of brick/block. We note that some on-going studies focus on classifying slums by the use of corrugated iron for roofs. This would not work in Nairobi, where over 50% of formal sector residential buildings also have corrugated iron sheet roofs (88% in slums).
We assume there is no amenity loss or gain from building tall so the rent of a unit of formal sector building volume, \( p_F(x, t) \), is exogenous to the developer, and is place and time specific. The present value of rent (per unit land) that accrues over the life of a structure, \( t \in [\tau_i, \tau_{i+1}] \), discounted to construction date \( \tau_i \) at interest rate \( \rho \) is denoted \( R_F(x, \tau_i) \). With costs \( k(v(x, \tau_i)) \) sunk and volume fixed at the date of construction this is given by

\[
R_F(x, \tau_i) = \int_{\tau_i}^{\tau_{i+1}} p_F(x, t)v_F(x, \tau_i)e^{-\rho(t-\tau_i)} \, dt - k(v_F(x, \tau_i)).
\]  

(5)

We define the present value of the rent of a unit of formal housing space over its life relative to rent at date of construction as

\[
\Phi(x, i) \equiv \int_{\tau_i}^{\tau_{i+1}} \left[ \frac{p_F(x, t)}{p_F(x, \tau_i)} \right] e^{-\rho(t-\tau_i)} \, dt,
\]  

so \( R_F(x, \tau_i) = p_F(x, \tau_i)\Phi(x, i)v_F(x, \tau_i) - k(v_F(x, \tau_i)) \). The integral \( \Phi(x, i) \) is akin to the ‘value-to-rent ratio’ on a newly constructed property in the terminology of the real-estate literature (noting the time horizon is cut at \( \tau_{i+1} \) in (6)).

The first order condition for choice of volume is

\[
\frac{\partial R_F(x, \tau_i)}{\partial v_F(x, \tau_i)} = p_F(x, \tau_i)\Phi(x, i) - k'(v_F(x, \tau_i)) = 0.
\]  

(7)

If the cost function is iso-elastic, \( k(v_F) = \kappa_F v_F^\gamma \), \( \gamma > 1 \), then the first order condition and the maximised present value of rent are,\(^1\)

\[
v_F(x, \tau_i) = \left[ \frac{p_F(x, \tau_i)\Phi(x, i)}{\kappa_F \gamma} \right]^{\frac{1}{\gamma-1}}, \quad R_F(x, \tau_i) = \kappa_F (\gamma - 1) \left[ \frac{p_F(x, \tau_i)\Phi(x, i)}{\kappa_F \gamma} \right]^{\frac{\gamma}{\gamma-1}}.
\]  

(8)

It is useful to have a continuous flow measure of land rent, given by amortizing the one-off construction cost continuously over the life of the structure. If amortization is constant proportion \( \mu \) of revenue then costs are fully covered by setting \( \mu \) to satisfy

\[
\mu p_F(x, \tau_i)\Phi(x, i)v_F(x, \tau_i) = k(v_F(x, \tau_i)).
\]  

With \( k(v_F) = \kappa_F v_F^\gamma \) and (8) the amortization rate is \( \mu = 1/\gamma \).\(^2\) Flow land rent, \( r_F(x, t, \tau_i) \), defined as gross revenue net of amortized costs, is therefore

\[^1\text{The iso-elastic form implies an elasticity of substitution between land and capital (i.e. construction cost) of unity. This is higher than many estimates in the literature, although at the centre of the range suggested in recent work by Ahlfeldt and Mcmillen (2014).}\]

\[^2\text{Using } k(v_F) = \kappa_F v_F^\gamma \text{ the condition } \mu p_F v_F \Phi = k(v_F) \text{ becomes } \mu p_F \Phi = \kappa_F v_F^{\gamma-1} \text{ and using the first eqn.}\]
Thus, flow land rent net of amortization is fraction \((1-1/\gamma)\) of revenue earned by land and structure together, while in the informal sector land rent is fraction \((1-1/\alpha)\) of revenue (eqn. 4). Notice that \(r_F(x,t,\tau)\) depends on location, time, and date of construction, and that the present value of rent over the lifetime of a formal sector building can be written as \(R_F(x,\tau) = \Phi(x,i)r_F(x,\tau,\tau)\).

2.2. Land development and construction phases

Continuing to focus on a particular unit of land, \(x\), we now look at the choice of when to develop (or redevelop) informal or formal structures. At some date (say time 0) the present value of rent from a unit of land at \(x\) that has not yet been developed is

\[
PV(x) = \int_0^{\tau_0} r_0 e^{-\rho t} dt + \int_{\tau_0}^{\tau_1} r_I(x,t) e^{-\rho t} dt + \left[ R_F(x,\tau) - D(x)\right] e^{-\rho \tau_I} + \sum_{\tau_{i+1}} R_F(x,\tau) e^{-\rho \tau_i}.
\]

The first term is the present value of rent from undeveloped land (flow rent \(r_0\) which we take to be constant), discounted at rate \(\rho\) and calculated up to the date of first development, denoted \(\tau_0\). The second term gives the present value of rent from informally developed land during interval \(\tau_0, \tau_1\). The first formal sector development, occurring at date \(\tau_1\) yields rent and incurs a one-time fixed cost \(D(x)\) of converting to formality. The final term in (10) gives the discounted value of rents earned over the lives of consecutive formal sector buildings, constructed at dates \(\tau_2, \tau_3\ldots\). The rent terms in this expression depend on house rents per unit volume \(p_I(x,t)\) and \(p_F(x,t)\) (eqns. (3) and (8)), which we assume to be monotonically increasing and exogenous.

The formalisation cost, \(D(x)\), captures the fact that formal sector development requires reasonably well defined property rights, such as land titling or a formal leasehold system like in Hong Kong. Obstacles to obtaining these rights on some properties may be substantial, particularly in African countries where much land is held traditionally under possessory and communal rights. \(D(x)\) includes the cost of obtaining formal title, which is highly variable even within a city depending on the history of the plot, as discussed later. We will later show that a substantial portion of slums especially nearer the city edge are on private (titled) land, so slums are not coincident with lack of land title. However slums nearer the city centre are

\[\text{in (8) to substitute for } v_F \text{ gives } \mu = 1/\gamma.\]

\[\text{To see this, take the ratio of eqns. (8), } R_F(x,\tau_i)/v_F(x,\tau_i) = p_F(x,\tau_i)\Phi(x,i)(\gamma - 1)/\gamma \text{ and use (9).}\]

\[\text{For simplicity, we do not let this depend on time. The dependence on location is drawn out in section 2.4}\]
classified as ‘government owned’ which will turn out to be a code word for being clouded by rights issues, political influence and corruption, which are costly to overturn.

Dates of development and redevelopment are chosen to maximise $PV(x)$. For the first development (which we assume for the moment to be informal), the optimal $\tau_0$ simply equates flow land rents on undeveloped and informal land, and is implicitly defined by

$$\frac{\partial PV(x)}{\partial \tau_0} = e^{-\rho \tau_0} \left[ r_0 - \rho \tau_0 \right] = 0. \quad (11)$$

The first formal development takes place at date $\tau_1$ satisfying

$$\frac{\partial PV(x)}{\partial \tau_1} = e^{-\rho \tau_1} \left[ p_F(x, \tau_1) - \rho k(x, \tau_1) \right] = 0. \quad (12)$$

(see appendix). A necessary condition for this to have an interior solution is that

$$0 < p_F(x, \tau_1) v_F(x, \tau_1) - \rho k(x, \tau_1) = p_F(x, \tau_1) v_F(x, \tau_1) \left[ y - \rho \Phi(x, i) \right], \quad \text{the second part of which uses} \quad k(x) = \kappa_F v_F'. \quad \text{and (8). We assume henceforth that this inequality holds.}$$

The first redevelopment of formal land is at date $\tau_2$ satisfying

$$\frac{\partial PV(x)}{\partial \tau_2} = e^{-\rho \tau_2} \left[ p_F(x, \tau_2) v_F(x, \tau_2) - p_F(x, \tau_2) v_F(x, \tau_2) + \rho k(x, \tau_2) \right] = 0, \quad (12)$$

(see appendix). Generalising this for all redevelopment gives:

$$p_F(x, \tau_{i+1}) [v_F(x, \tau_{i+1}) - v_F(x, \tau_i)] = \rho k(x, \tau_{i+1}), \quad \text{for} \quad i \geq 1. \quad (13)$$

This condition says that demolition and reconstruction occur at the date at which the revenue gain from the change in volume equals the interest cost of the construction expenditure incurred. Similar intuition applies to eqn. (12).

Equations (11) – (13) implicitly define the dates at which sites are (re-)developed. Using the optimised values of $v(x, t)$ given by eqns. (3) and (8), the house rents that trigger development and hence the dates of development are given by the following eqns. (11a) – (13a). The date at which site $x$ becomes informally developed, $\tau_0$, is implicitly defined by

$$p_F(x, \tau_0) = \frac{\kappa_F \alpha}{d_0} \left[ \frac{r_0}{(\alpha - 1) \kappa_F} \right]^{\left(1/\alpha\right)}. \quad (11a)$$

The right hand side of this expression is constant, and can be thought of as giving a trigger value; location $x$ becomes informally developed on the date at which house rent at $x$ reaches this trigger level.
The date at which informal settlement becomes formalised, $\tau_1$, is given by eqn. (12) which using (3) and (8) becomes

$$\kappa_i (\alpha - 1) \left[ \frac{d_x p_f(x, \tau_i)}{\kappa_i \alpha} \right]^{\alpha - 1} = \kappa_F \left( \frac{\gamma}{\Phi(x, l)} - \rho \right) \left[ \frac{p_f(x, \tau_i) \Phi(x, l)}{\kappa_F \gamma} \right]^{\gamma - 1} - \rho D(x).$$

(12a)

The dates at which successive formal redevelopments of $x$ take place, $\tau_i, i > 1$, become

$$\left[ \frac{p_f(x, \tau_i) \Phi(x, i)}{p_f(x, \tau_{i+1}) \Phi(x, i+1)} \right]^{\gamma - 1} = \frac{\gamma - \rho \Phi(x, i + 1)}{\gamma}. \quad (13a)$$

These three equations, (11a)-(13a) together with the definition of the value-to-rent ratio, $\Phi(x, i)$ in eqn. (6), form the basis of the analysis of the next sub-section.

2.3. Analysis

What do we learn from the characterisation of development stages given above? A benchmark case in which house rents are growing at constant exponential rates, $\hat{p}_i, \hat{p}_F > 0$ yields analytical results. The full general equilibrium model that supports constant exponential price growth is discussed in section 2.6 and detailed in the Theory Appendix, but for the present we simply assume these house rent paths. We look at the time series development of a particular place, $x$, and then at the urban cross-section.

**Urban dynamics:** To draw out results we look first at successive redevelopments of formal areas of the city, and then turn to the city edge and informal development.

**Proposition 1:** If formal sector construction costs are iso-elastic in height (with elasticity $\gamma$), informal sector quality is iso-elastic in cover (with elasticity $(1 - \alpha)/\alpha$), prices are growing at constant exponential rates $\rho > \hat{p}_i, \hat{p}_F > 0$, and agents have perfect foresight then:

(i) The value-to-rent ratio takes constant value $\Phi$, and the time interval between successive formal redevelopments is constant $\Delta \tau$,

$$\Phi = \int_0^{\Delta \tau} e^{(\hat{p}_F - \rho)t} dt = \frac{1 - e^{(\hat{p}_F - \rho)\Delta \tau}}{\rho - \hat{p}_F}, \quad \Delta \tau = \frac{(\gamma - 1) \ln \left[ \frac{\gamma}{\gamma - \rho \Phi} \right]}{\hat{p}_F}. \quad (14)$$

(ii) Successive rounds of formal sector building have greater volume (height) by a constant proportional factor.
\[ \frac{v_F(x, \tau_{i+1})}{v_F(x, \tau_i)} = e^{\frac{p_F \Delta \tau}{(\gamma - 1)}} = \frac{\gamma}{\gamma - \rho \Phi}. \]  

(iii) If the rate of growth of prices is the same in all locations, \( x \), then so too are \( \Phi \), \( \Delta \tau \), and volume growth.

The first part of this proposition comes from integrating eqn. (7), using it in (13a), and noting that there is a unique solution solving the two parts of (14) with constant \( \Phi \) and \( \Delta \tau \). The second part follows by using this in the first order condition for volume, (8). The third comes from noting that (14) and (15) do not depend on \( x \). While volume ratios and time intervals do not vary with \( x \), the actual dates of redevelopment do, as discussed below.

What about the earlier stages of informal development? The first transition we assumed is from agriculture to informal settlement. This occurs for land at \( x \) when \( p_F(x, t) \), the quality un-adjusted informal sector house rent, reaches the trigger value given by (11a). A period of informal settlement exists only if the return to informal settlement at date \( \tau_0 \) is greater than commencing formal settlement, \( r_i(x, \tau_0) > p_F(x, \tau_0)v_F(x, \tau_0) - \rho \{k_F(v_F(x, \tau_0)) + D(x)\} \) (see eqn. (12)). If not, then initial development will be formal, with date \( \tau_1 \) implicitly defined by \( r_0 = p_F(x, \tau_1)v_F(x, \tau_1) - \rho \{k_F(v_F(x, \tau_1)) + D(x)\} \).

The transition from informal to formal settlement is given by date \( \tau_i \) that solves (12a). There is a unique transition date satisfying the second order condition if the return to formal development is rising faster than the return to informal settlement (i.e. the right hand side of (12a) is increasing faster than the left). If \( D = 0 \), a necessary and sufficient condition for this is that \( \hat{p}_F \gamma (\gamma - 1) > \hat{p}_F \alpha (\alpha - 1) \). If \( D > 0 \), then this condition is sufficient but not necessary. We assume the condition to be satisfied, as it will be if house rents (before being deflated for crowding) increase at the same rate and \( \alpha > \gamma \). The condition \( \alpha > \gamma \) means that the share of land rent in revenue is higher (and the share of construction costs lower) in informal development than in formal (eqns. (4), (9)). It also means that the elasticity of land rent with respect to house rent is lower in informal sector housing than formal (compare eqns. (3) and (8)). Essentially, there are sharper decreasing returns to increases in volume in the informal sector (where crowding reduces house rents) than in the formal sector (where building taller raises unit construction costs). Our empirical work suggests that the condition is satisfied.

Figure 1 pulls these stages together and illustrates the development path. Parameter values are based on the empirical work in Section 3. They are reported in the appendix and derived from calibration discussed in Section 2.5. Building volume is given on the vertical axis (log units), and on the horizontal plane axes are time \( t \) and location \( x \). Location is distance from the CBD, and we discuss the cross-section – variation across \( x \) at a given \( t \) – in the next sub-
section. For the moment, look just at the development of a particular location through time, i.e. fix \( x \) and look along a line sloping up and to the right with the \( t \) axis. Initially (at low \( t \)) this land is rural. Building volume becomes positive at date \( \tau_0 \) (specific to location \( x \)) when informal development takes place. The volume of informal development increases steadily, as increasing \( p_f \) causes Meccano pieces to be rearranged and building cover to increase. Formal development takes place at \( \tau_1 \) and, as illustrated, leads to an increase in volume, indicated by the second step. However, the relative change in volume depends on parameters, and it is possible that edge slums deliver more volume than does first stage formal development. Subsequent redevelopments occur at fixed time interval \( \Delta \tau \) and bring the same proportionate increase in volume, achieved by building taller. The timing and volume of each of these formal investments is based on perfect foresight about the growth of prices and the date of subsequent redevelopments.

The urban cross-section. We have so far concentrated on a single location, \( x \), and now show how development depends on \( x \). Henceforth, \( x \) is interpreted as distance from the CBD, and house rents decrease with distance. We assume exogenous rates of decrease at exponential rates \( \theta_f \), and the appendix derives these from commuting costs. Exponential decline with respect to distance together with exponential growth through time mean house rents are

\[
p_I(x,t) = \bar{p}_I e^{\beta t} e^{-\theta_I x}, \quad p_F(x,t) = \bar{p}_F e^{\beta t} e^{-\theta_F x}.
\]

Thus, the trigger house rent for informal development in eqn. (11a) depends on both date and place according to

\[
p_I(x,t) = \bar{p}_I e^{\beta t} e^{-\theta_I x} = \frac{\kappa_I \alpha}{a_0} \left[ \frac{r_0}{(\alpha - 1)\kappa_I} \right]^{1/\alpha}.
\]

This can be interpreted either as giving the date at which place \( x \) develops or the place that develops at date \( t \), i.e. the date \( t \) city edge, \( x_0(t) \). Similarly, equation (12a) can be interpreted as giving the distance at which first formalisation occurs at date \( t \), \( x_1(t) \), and (13a) the distances of successive redevelopments occurring at date \( t \), \( x_i(t) \), \( i > 1 \).

On Figure 1 the urban cross-section is given by fixing a date and moving along a line parallel to the \( x \) axis, with steps in volume occurring as illustrated. At the city edge land is informal and, moving towards the centre, locations that have been urban for longer have been through more stages of development and offer greater building volume per unit land. The increase in volume is achieved by increasing land cover in the informal area and by greater height in formal areas closer to the centre.
Putting the parts together, we see how the urban cross-section evolves through time. Proposition 2 states results on how different stages of development (building types and heights) move across the city as it grows.

**Proposition 2:** If formal sector construction costs and informal sector quality are iso-elastic, rents are growing at constant exponential rates \( \rho > \hat{\rho}_I \), \( \hat{\rho}_F > 0 \) and declining with distance at constant rates \( \theta_I, \theta_F > 0 \), and agents have perfect foresight then:

(i) The distance from the city centre to the edge of informal development increases through time according to \( \frac{dx_i}{dt} = \hat{\rho}_I / \theta_I \).

(ii) If \( D(x) = 0 \), the distance from the city centre to the edge of formal development increases through time according to \( \frac{dx_f}{dt} = \hat{\rho}_F \gamma (\alpha - 1) - \hat{\rho}_F \alpha (\gamma - 1) \).

(iii) The distance between successive formal sector redevelopments, \( \Delta x \), is constant,

\[
\Delta x = \frac{(\gamma - 1) \ln \left[ \frac{\gamma}{\gamma - \rho \Theta_F} \right]}{\Theta_F}. \tag{17}
\]

Part (i) is simply the total differential of (11b) with respect to \( x \) and \( t \). Part (ii) comes from differentiation of (12a) which implicitly defines the place at which first formal development occurs at date \( t \), together with price equations (16), and the fact that \( \Phi \) is independent of \( x \) and \( t \), as given in proposition 1. For Part (ii) we explore the implications of \( D(x) > 0 \) in the next sub-section. Part (iii) of the proposition follows from eqns. (13a), noting that the price ratio on the left-hand side of (13a) now compares prices at different \( x \) and the same \( t \),

\[
p_F(x_i(t), t) / p_F(x_{i+1}(t), t) = e^{-\Phi \Delta x}, \text{ where } \Delta x = x_i(t) - x_{i+1}(t), \text{ i.e. the distance between places undergoing successive redevelopments.} \tag{5}
\]

As expected, comparison of (17) with (14) gives \( \Delta x / \Delta t = \hat{\rho}_F / \Theta_F \), this indicating how the prices that trigger redevelopment relate across space and across time.

Figure 1 combines the urban time path and cross-section, and is constructed with \( \hat{\rho}_I = \hat{\rho}_F \) and \( \theta_I = \theta_F \) so the lines along which development and redevelopment occur are parallel, as implied by proposition 2. It follows that the width of the informal area, \( x_1 - x_0 \), is constant through time. Hence one can show that, even in a circular city, the share of urban land area that is informal falls with time and as the city gets larger. Generally, the area of land occupied by the informal sector becomes narrower through time if house rent growth is faster.

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5 Whereas in proposition 1, the price ratio in equation (13a) was evaluated at given \( x \), so \( p_F(x, \tau_i) / p_F(x, \tau_{i+1}) = e^{-\hat{\rho}_F \Delta \tau} \).
in the formal sector than informal (quality unadjusted) or the formal sector house rent
gradient is flatter than that of the informal sector, \( \dot{p}_F / \theta_F > \dot{p}_I / \theta_I \).\(^6\)

While our analytical results are based on constant exponential price paths, we note that it is
also possible to numerically compute the perfect foresight equilibrium for more general rent
paths.

2.4. Frictions and the cost of imperfections

The analysis so far has concentrated on a benchmark case, with Proposition 2 assuming no
formalization costs. We now consider heterogeneity across places where formalisation costs,
\( D(x) \), exist and may vary with \( x \). Second, results so far have assumed perfect foresight; we
relax this, and look briefly at the implications of systematic deviations from perfect foresight.

**Formalisation costs.** Locations vary in their distance to the city centre and, now also in their
formalisation costs, \( D(x) \geq 0 \). We suppose that these costs may be due to institutional rather
than real costs, creating inefficiency in the equilibrium outcome. Figure 2a illustrates the
implications of there being an interval of \( x \) within which \( D(x) \) is positive (and zero
elsewhere). As expected, this extends the period during which the area is occupied by
informal settlement. A persistently informal area will see housing volume per unit area
increase through time as informal buildings are reshaped and crowding increases. It is
possible that it may come to have volume higher than the surrounding formal area, as
illustrated in figure 2a and something we will see in the empirics; however, additional
informal volume is achieved by crowding, not by height.

A history of informality has a persistent legacy on the area. Formal development starts later,
and so therefore does subsequent redevelopment, as illustrated, which impacts building
volume as driven by the price (and hence date) at which redevelopment occurs (eqn. 8).
Proposition 1 still holds for the time series evolution of each place, meaning \( \Phi \) and \( \Delta \tau \) are
still the same across time and space for all formal sector developments. But because of
different start dates, at any \( x \) in the urban cross-section, there can now be enormous variation
in building volume and height within just the formal sector. This is illustrated vividly in
figure 2b, in which \( D(x) \) is set at random non-negative values across space. All locations see
volume increase with time, but initial and subsequent formal development takes place at
different dates and builds to different heights. This means, for example, that gradients of
volume for existing buildings may no longer be monotonically decreasing from the centre in
such a city and there will be heterogeneity by ray from the city centre. Patterns are the

\(^6\) The general expression is

\[
\frac{dx_i}{dt} - \frac{dx_0}{dt} = \frac{\gamma(\alpha - 1)(\dot{p}_F \theta_F - \dot{p}_I \theta_I)}{\theta_I \{\theta_F(\alpha - 1) - \theta_I(\gamma - 1)\}}.
\]
hotchpotch we see in the data. However, from eqn. (8), any new developments in the formal sector at a time \( t \) will experience the same monotonic decline in volume as we move from the city centre, since they follow housing rents which decline with distance from the centre. This will be key to our calibration of the model.

What are the welfare implications of this hotchpotch of different land use? Under an open city model in which time paths of residents’ utility are given exogenously, welfare implications are captured entirely in rents going to land owners. In Section 4 we measure the rent and land value foregone from artificially high formalisation costs for older slums in Nairobi.

**Expectations:** We have assumed, so far, that decisions are based on perfect foresight. What are the consequences of removing this assumption? Recall that \( \Phi(x,i) \) is the value-to-rent ratio on a newly constructed property over the life of the building, and eqns. (14) give the perfect foresight values of this and of the expected length of life of the property, \( \Delta \tau \). How do results change if construction decisions are based on a value-to-rent ratio that differs from the perfect foresight ratio? For the parameters used in Figure 1 the perfect foresight value-to-rent ratio is \( \Phi = 25.7 \), and the interval between redevelopments is \( \Delta \tau = 65.6 \). We conducted a numerical experiment in which developers have less positive expectations and build on the basis of a value-to-rent ratio of 19.3 (imposed at 75% of the perfect foresight value). The transition from rural to informal settlement is unaffected by this, but formal development is based on these less optimistic expectations. As a consequence, developers build less volume and hence buildings become obsolete more rapidly, so the interval between redevelopments drops to \( \Delta \tau = 38 \).

The welfare cost of this imperfection is measured by its impact on land rents. We compute the present value of these rents, integrating over the locations and dates illustrated in figure 1 (i.e. out to \( t = 180 \) and to distance 50). Lower expectations reduce the present value of land rents by 26%.\(^7\) This is a substantial amount, noting that, under the scenario, city population is smaller which would create further losses if there were urbanisation economies and/ or a wedge between urban and rural marginal products of labour.

### 2.5 Cross-section predictions and calibration

The theory implies that building volumes and land rents decline with distance from the CBD. In the informal sector the spatial gradients of both volume and land rents are, from eqns. (3) and (16), \(-\partial_{\alpha}/(\alpha - 1)\), as reported in the first column of Table 1. They are illustrated in Fig. 3 which has distance from the city centre on the horizontal axis and the log of volume (panel

\(^7\) As a percentage of the excess of urban land rent over the rent earned by land in non-urban use, \( r_0 \).
a) and land rents (panel b) on the vertical. The figures correspond to the cross-section of Fig.1 at $t = 140$, with land informally developed in the interval $[x_0, x_1]$ and height and volume indicated by the solid lines. The dashed lines extend this, giving what volume and rent would be at current house rents if informal development extended closer to the centre.

The second column of Table 1 and the kinked lines in Fig. 3 give volume and land rent in the formal sector. This sector has volume fixed within each stage of development and discretely lower at development stages further from the centre. The volume gradient between points of redevelopment is

$$\frac{dv_F}{dx} \frac{1}{v_F} = \frac{\Delta \ln(v_F)}{\Delta x} = -\frac{\theta_F}{(\gamma - 1)} \text{, from eqns. (15) and (17).}$$

Panel B of Fig. 3 illustrates land rent net of amortization of building costs, $r_F$. This declines with distance, with discrete reductions at each point of redevelopment and a gradient between these points of $-\frac{\theta_F \gamma}{(\gamma - 1)}$, from either eqns. (8) and (9).

**Table 1: Spatial gradients**

<table>
<thead>
<tr>
<th></th>
<th>Informal:</th>
<th></th>
<th>Formal:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volume</strong> (per unit area)</td>
<td>$\frac{dv_i}{dx} \frac{1}{v_i} = -\frac{\theta_i \alpha}{\alpha - 1} = -0.0902$</td>
<td>Inter-$i$: $\frac{\Delta \log(v_F)}{\Delta x} = -\frac{\theta_F}{\gamma - 1} = -0.0924$</td>
<td></td>
</tr>
<tr>
<td><strong>Land rent</strong> (per unit area)</td>
<td>$\frac{dr_i}{dx} \frac{1}{r_i} = -\frac{\theta_i \alpha}{\alpha - 1}$</td>
<td>Inter-$i$: $\frac{\Delta \log(r_F)}{\Delta x} = -\frac{\theta_F \gamma}{\gamma - 1} = -0.155$</td>
<td></td>
</tr>
</tbody>
</table>

The gradients given in Table 1 contain four parameters $\{\alpha, \gamma, \theta_I, \theta_F\}$. From the Nairobi data (which we discuss in detail in section 3) we observe building volume for formal and informal development, based respectively on the height of new formal buildings and the cover to area ratio in slums. The estimated slope coefficients of these volumes with respect to distance from the centre give the numbers in the first row of Table 1. We also have data on the price of vacant land in the city from which we estimate its spatial gradient. Since development of such land is entirely formal and the ratio of land price to land rent is constant this gives the gradient of formal land rent, in the second row and column of Table 1. Given these numbers, we calibrate parameters by assuming that $\theta_I = \theta_F$ and using the equations in the table to derive values $\alpha = 3.33$, $\gamma = 1.68$ and $\theta_I = \theta_F = 0.063$, used in the analysis of Figures 1-3.

Several comments are in order. First, we have a further source of Nairobi price data which is consistent with the numbers above. Survey data on house rent (per square meter) give a

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8 Fig.3 illustrates the case with zero formalisation costs.

9 The regressions from which these numbers are derived are discussed in Section 3. Volume gradients are Table 3a column 2, 3b column 3, and land price gradients Table 2 column 1.
house-rent gradient, $\theta_F$, very close to the calibrated value $\theta_F = 0.063$. This is discussed further in section 3.3.

Second, the reality is a hotchpotch. The volume gradients are derived by looking at the height of redeveloped buildings in the city, which will occur at almost all distances from the centre. As noted in Section 2.4, the heights of redeveloped buildings are still governed by eqn. (8) regardless of where they occur in the current period. So differentiating eqn. (8), we get the same slope as in Table 1 column 2. Similarly for land prices. Vacant lots are indicative of land prices for land about to be redeveloped.

Third, as discussed earlier, the relative gradients of formal and informal sector land rents determine which type of development is closer to the city centre. With $\theta_I = \theta_F$ formal is closer to the city centre if $\alpha > \gamma$, as is the case with our calibrated values. The calibrated value of $\gamma = 1.68$ implies that the share of land rent in formal sector revenue is about 40%, while $\alpha$ implies a corresponding share in the informal sector of 70% (eqns. (9) and (4) respectively). The formal sector share is in line with developed country data (Ahlfeldt and McMillen 2014, Duranton and Puga 2015). For slums there are no data we know of to make comparisons, since slum land is generally not officially transacted. However, given the low construction costs and land intensity of slum housing such a high rent share seems reasonable.

2.6 Closing the model

To this point our analysis has posited given time paths for the rent of housing volume of each type at each location. The model can be completed by specifying household behaviour, commuting and hence the demand for space at each place and time. This is constructed in a way consistent with the preceding analysis, offering a model of price and population growth arising as city productivity grows faster than productivity outside. Technical exposition is in the Theory Appendix and assumes consumers have log linear preferences and commuting costs such that income net of commuting costs declines exponentially with distance from the centre. The city is open with free migration from outside.

3. Empirical work on Nairobi

The empirical work provides the evidence we use to calibrate the model, details the dynamic development of the built environment of Nairobi and, examines the specific policy issue of the costs of land market imperfections. We present cross-section patterns of land sale prices, house rents, building heights, coverage and volume gradients. What is novel is distinguishing the role of the slum versus the formal sector to show how patterns differ between the two over a whole city, as well as each sector’s overall contribution to the city’s built stock. The
dynamics uses building footprint polygons from high resolution data for 2003/4 and 2015 and building heights for 2015. We derive the changes over the 11 or so years in height, cover, and volume overall and within the slum and formal sector. The volume changes indicate a city in rapid evolution in both the slum and formal sectors.

For slums, we ask if they move away from the centre and spring up on the edge, and whether their role is shrinking or rising. Key to the last question is the policy issue. What is the role of formalisation costs? For Nairobi, based on “accidents” of history, we have an empirical counterpart: slum settlements where formalisation barriers are high, driven by politics. We calibrate the welfare costs of the inability to convert slum lands to their highest and best use.

In this section we first describe the data in more detail and then present results on the cross-section and dynamics of the city.

3.1 Data and mapping

We develop a data set for Nairobi which defines characteristics of the built environment at a very fine spatial resolution. Characteristics are defined at no more than 40 cm resolution and, based on that, mapped for 3m x 3m cells and aggregated preserving details to a grid of 150m x 150m. For the sample we focus on in the next two sections, the intensive margin of the 2003/4 built area of the city, there are 6470 such grid cells.

Our main data consist of building footprints based on tracings of buildings from aerial photo images for 2003/4 and 2015. The key methodological imagery work has been to overlay the 2003/4 polygons with those for 2015 to determine which building footprints are unchanged since 2003/4, which buildings were demolished and/or redeveloped and finally where and to what extent infill occurs. Overlay is complicated by variations in the way buildings were traced and aligned in 2004 versus 2015. The Data Methodology Appendix describes the issue and the algorithm used to overlay and identify types of changes.

We also have building height data for 2015 from LiDAR (0.3-1m resolution) which was used to create a Digital Elevation Model. For 2003/4 heights, we assume the height of unchanged buildings is the same as in 2015 (ignoring the possibility of adding floors to a structure). For demolished buildings in a grid square, for their 2003/4 height, we assign the average height of unchanged buildings in that sector in the 8 queen neighbouring grid squares. Both

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10 We received the 2003/4 footprint data from the Nairobi City Council with digitized polygons for every building in the administrative boundary of Nairobi, as based on documentation from the Center for Sustainable Urban Development (CSUD) at Columbia University, who use a highly detailed land-use map from the Japan International Cooperation Agency (JICA). As far as we can tell, this data was created by JICA and the Government of Kenya under the Japanese Government Technical Cooperation Program, and mostly based on aerial images taken in February 2003 at a scale of 1:15,000 (Williams, et al. 2014). In January 2015, imagery at (10-20cm resolution) was recorded and digitized into building footprints by Ramani Geosystems who also provided the LiDAR data from which building heights are extracted.
inferences are likely to overstate average 2003/4 heights and thus understate volume changes. Demolished buildings are likely less tall than unchanged ones as suggested by the model. We use high resolution SPOT satellite data for the years 2004 and 2013 to measure road coverage.

For Nairobi we utilize two classifications of slums. One is a 2003/4 land use map prepared by the CSUD at Columbia University.\textsuperscript{11} The other, on which we rely more heavily, is a 2011 slum mapping by IPE Global Limited under the Kenya Informal Settlements program. IPE mapping of settlements was done using satellite imagery and topographic maps with the general idea that slums are “unplanned settlements” which have some aspects of low house quality, poor infrastructure, or insecure tenure. The 2011 designation has many more slums than in 2003/4. Some 2011 areas nearer the city edge had housing in 2003/4 not then defined as slums; in most cases these areas subsequently experienced enormous contiguous development of small densely packed buildings. It is clear that the effective definitions differ in detail across years and cannot be used to distinguish slum creation and destruction per se.

In Fig. 4 we show these two mappings of slums and define the area of the city we work with. We adopt a fairly conservative definition of the urban boundary based on built area. For a (150mx150m) grid cell to be within the city, the average roof cover in cells whose centroid is within a 900 meter radius of the cell must be above 10%; and we only keep those cells which contiguously connect to the CBD. Fig. 4 shows the city in dashed outline in 2003/4 and in solid outline in 2015. For each year we mark the slums as recorded at that time and in both periods: green if in both years, yellow if only in 2011 and blue if only in 2003/4. We mark the radius in red near the CBD in which there are no slums as defined in each time period (dashed and solid) and the city centre with a yellow star. The city centre is the brightest lit pixel in night lights data in the early 1990’s.

We focus on the intensive margin which is the 2003/4 city, but at points discuss the significant sized extensive margin. Nairobi is far from circular, being bounded to the south by an airport and a large national park and to the immediate north of the centre by a preserved state forest. Slums are not prevalent near the centre; and the area with no slums as defined contemporaneously expands considerably between the two years, from a 0.775 km to a 2.0 km radius around the centre by 2011. The map suggests considerable slum expansion (yellow) at the 2004 fringe of the city and beyond, as predicted in the model. However, the maps suggest there has been little slum removal (blue). Finally, we note the large slum of

\textsuperscript{11} Center for Sustainable Urban Development (CSUD) at Columbia University, and based on a more detailed, copyrighted, landuse map created by the JICA and the Government of Kenya under the Japanese Government Technical Cooperation Program which was published and printed by the survey of Kenya 1000 in March 2005. See Williams, et al. 2014 for the full methodology. In principle, polygons are categorized as slums if they seemed to contain small mostly temporary buildings that are randomly distributed in high density clusters.
Kibera directly south-west of the centre (ranging from 3-5 km of the centre). In section 4, we will discuss Kibera and other slums near the centre.

In Fig. 5 we show a 3-D map of the city for the 2003/4 boundary, which gives the average height of all buildings in public or private use in each grid square (assigned to slum or formal sector by where the centroid of the grid square lies). Calculations are discussed below and details are in the Appendix. Blank areas are those which have censored data in 2004 (e.g. Moi airbase) and large areas that have no cover such as the Kibera golf course. The city does look monocentric with tall but variable height at the centre and then diminishing. Slum areas in red are generally low height. In the north-east they also reveal misclassification problems; satellite images indicate that some tall red areas are not slums.

For the empirical analysis we adjust the areas of analysis in Figs. 4 and 5 in two ways. Sectoral classification focuses on slums, and does so with very tight boundaries cutting off vacant land adjacent to the slum (including roads or rivers dividing a slum) and even edge slum housing. The formal sector is a residual of everything else in the city. To do a proper comparison we first remove all grid squares entirely in permanent public use (or not traced in 2004) amounting to 11% of land in the 2004 city boundary, and 25% at the centre (0-1 km, including parks and the Presidential palace). Note that neighbourhood schools and all roads are not removed. The second issue is the tight delineation of slum areas. To offset this, we adjust the IPE slum boundaries by first classifying buildings as slum if their centre lies within the original slum boundary, and then assigning each 3m x 3m pixel of non-built land to slum if the nearest building is classified as slum, and formal otherwise.

3.2 Defining the built features of a city in the cross-section

To analyse the built environment and the dynamics of change we define some key concepts and a basic decomposition of the sources of building volume in a city.

Each cell (3x3m) is classified as either informal/slum (I) or formal (F) on the basis of the adjusted IPE map. These cells are then aggregated up to 150m x 150m grid squares. As noted earlier we removed grid squares that are in public use, so what is left is just slum and formal. We have the following definitions: $a_i(\chi)$ is defined as the area (m$^2$) of grid square $\chi$ that is occupied by type $i$, $i = I, F$ (as defined by the binary classification at the 3x3m level). The total area is $a_i(\chi) + a_f(\chi) = 22500$. $c_i(\chi)$ is the type $i$ covered area, or total of building footprints in type $i$ areas of grid square $\chi$ (in m$^2$). $\bar{h}_i(\chi)$, is the average height (based on 3x3m cells) of the covered area $c_i(\chi)$. Finally, $v_i(\chi) = \bar{h}_i(\chi) c_i(\chi)$ is the total volume of built space of type $i$ in the grid square (in m$^3$).

Much of our analysis is based on distance $x$ from the city centre. We define the area at $x$ as all grid squares $\chi$ within a ring at $x$, i.e. $\chi \in x$. Thus, the total area of type $i$ at distance $x$ is
\[ a_i(x) = \sum_{\chi \in x} a_i(\chi), \text{ the total type } i \text{ building footprint at } x \] is \[ c_i(x) = \sum_{\chi \in x} c_i(\chi) \text{ and total volume} \]

supplied is \[ v_i(x) = \sum_{\chi \in x} v_i(\chi). \] For each distance \( x \) we define the ‘cover area ratio’ (CAR) by

type of use, \( CAR_i(x) = c_i(x) / a_i(x) \). To measure building volume, we use a new concept: the ‘built volume to area ratio’ (BVAR) where \( BVAR_i(x) = v_i(x) / a_i(x) \). This is like a floor to area ratio except that it is in cubic meters of space, related to floor space by dividing by average height of 3-3.1 m. For both CAR and BVAR the area is not lot size but all unbuilt land which includes side streets, vacant lots, and small public use. Average height of built space is \( \bar{h}_i(x) = \sum_{\chi \in x} \bar{h}_i(\chi)c_i(\chi) / c_i(x) \).

For each \( x \) we denote the share of area in slums as \( \rho_i = a_i(x) / \sum_i a_i(x) \) and the share in formal as \( 1 - \rho_i \). For total volume at \( x \), \( v(x) = \sum_i v_i(x) \), we have a decomposition:

\[
v(x) = \frac{a(x)}{\text{total area}} \left\{ \frac{\rho_i}{\text{share slum}} \left[ BVAR_i(x) + (1 - \rho_i) \cdot BVAR_f(x) \right] \right\}
= \frac{a(x)}{\text{total area}} \left\{ \frac{\rho_i}{\text{share slum}} \left[ \bar{h}_i(x) \cdot CAR_i(x) + (1 - \rho_i) \cdot \bar{h}_i(x) \cdot CAR_f(x) \right] \right\}
\]

(18)

### 3.3 Nairobi in the 2015 cross-section

We now use the data and our analytical structure to draw out the facts about Nairobi and derive the relationships used to calibrate the model. We display a series of graphs and regressions, looking first at the cross-section (land prices, house rents, and built fabric), and then at change in section 3.4.

**Vacant land prices:** For 2015, we have vacant-land prices, obtained by scraping from property24.co.ke in 2015. This website advertises property for sale in Kenya. We utilize vacant land listings with information on asking price and plot area.\(^{12}\) Listings are only found for the formal sector. Table 2 shows estimates of the key price gradients as defined in Section 2 and Table 1. Our calibration uses column 1 of Table 2, which gives the slope of the gradient for per square meter asking prices, which we take as corresponding to the present value of future land rents, as presented in the theory section.\(^{13}\) We have basic controls to standardize land for sale (see table footnote). The gradient is well represented by an

\(^{12}\) 80% of the listings have this information.

\(^{13}\) By focusing on slope not height we mitigate the issue of there being a mark-up in asking prices.
exponential relationship (eqns. 8-10 and 11b), where each km of further distance from the centre reduces price by 15.5% as shown in column 1 of Table 2 and illustrated in Fig. 6.

<table>
<thead>
<tr>
<th>Table 2. Price gradients</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ln PV(x): asking price per sq m. for vacant land for x &gt; 2km</td>
<td>-0.155***</td>
<td>-0.0884***</td>
<td>-0.0512***</td>
</tr>
<tr>
<td>Distance to centre</td>
<td>(0.0157)</td>
<td>(0.0176)</td>
<td>(0.0173)</td>
</tr>
<tr>
<td>Ln rent(x) per sq m. per month floor space for x &gt; 2.5 km</td>
<td>0.136***</td>
<td>0.0659**</td>
<td></td>
</tr>
<tr>
<td>Distance to centre x Slum =1</td>
<td>(0.0318)</td>
<td>(0.0274)</td>
<td></td>
</tr>
<tr>
<td>Slum=1</td>
<td>-1.228***</td>
<td>-0.463***</td>
<td></td>
</tr>
<tr>
<td>Controls on house (#22) and neighbourhood (#2)²</td>
<td></td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Controls: lot size, listing month and estimated coordinates</td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>10.45***</td>
<td>6.147***</td>
<td>5.695***</td>
</tr>
<tr>
<td></td>
<td>(0.0758)</td>
<td>(0.131)</td>
<td>(0.306)</td>
</tr>
<tr>
<td>Observations</td>
<td>544</td>
<td>983</td>
<td>902</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.585</td>
<td>0.132</td>
<td>0.37</td>
</tr>
</tbody>
</table>

1 Units are 2012 Kenyan shilling and km.
2 Written tenancy agreement, piped water in compound, 1 bathroom, 2+ bathrooms, share house, room in house, single-storey shared facilities, shack, multi-storey private bath, multi-story shared bath, walls brick/block, walls mud/wood, walls mud/cement, walls only wood, walls corrugated iron sheets, walls tin, floor tiles, floor cement, flooded once last rainy season, flooded 2-3 times, flooded more than 3 times, ln number floors, EA building cover (CAR), EA building density.

**House rents:** We have a georeferenced household level data set from the 2012 ‘Kenya: State of the Cities’ survey by the National Opinion Research Center (NORC). These are the first data set to record household rent per square meter (with detailed house and some neighbourhood characteristics) in Nairobi for a sample that is stratified between slum and formal areas (based on the 2009 Census).¹⁴ We estimate house rent gradients using a hedonic regression on NORC data of monthly rent per sq meter of floor space, after factoring out quality differences, to get a ‘pure’ distance premium for the formal sector. Controls include 22 variables (see footnote to the table) characterising house attributes and living conditions and two measures of crowding, as well as a control for slum and slum distance from centre to deal with unobserved conditions in slums. The estimated gradient slope for formal sector rents of -0.0512 is in column 2 of Table 2, very close to what we use in calibration (-0.063). This estimated slope is just for residential housing and is somewhat sensitive to specification, which is why we rely on column 1 estimates. Note that the slum gradient for observed rent is almost flat (slope of -0.051 + 0.066), as was assumed in the modelling.

¹⁴ Prior studies look just at slums (e.g. Guylani and Talukdar, 2008) and so offer no comparison across sectors.
We now turn to Table 3 and a series of figures which show built volume at different distances by slum and formal, and its decomposition by height and cover, as indicated by eqn. (18).

**Building heights:** Fig. 7a gives average building height, $\bar{h}(x)$, in each sector by distance from the centre. In the formal sector, this declines sharply from almost 30m at the centre until levelling out at about 7-8m. These are smoothed curves for grid squares whose centroid is in a 300m moving window going out from the centre. The gradient slope is given in Table 3a column 1 for all buildings and in column 2 for redeveloped buildings. As explained in section 2.5, following the model, we use the gradient for redeveloped or new buildings in calibration. In the slums in Fig. 7a and in Table 3b columns 1 and 2, height is flat at under 5m throughout, as assumed in the technology modelling in section 2. The building materials of slum housing do not permit building high. These relative gradients also hold for heights in just the residential sector by floors in the formal and slum sectors from the NORC data.

Fig. 7b shows the variability of height in meters within sector. Especially near the centre in the formal sector there is large spikiness or variability. While we have combined office towers, historical buildings, and housing, much of the variability could come from the differential timing at any location on when a property became formalized and could be built high, based on the specific history of the property’s path to formalization, as discussed in Section 2.4. In slums that issue does not exist, so that, especially in the older slums from 3-6 km out there is little variability. The modest increase in variability further out may reflect misclassification issues noted in the discussion of Fig. 5.

Some Africa experts we talked to seemed to believe that African cities were generally built without height, with buildings limited to 5 story walk-ups because, for example, of unreliability of power for elevators. Nairobi does not fit this description. Overall, buildings from 0-1 km of the centre average (at the 3m x 3m pixel) 10 stories (at 3.1m a storey) and in Fig. 7c, 5% of these pixels are over 16 stories.

**Coverage to area [CAR]:** Next we look at the cover to area ratio, CAR. In the formal sector, column 3 of Table 3a and Fig. 8 show that CAR is basically constant in the formal sector as we assumed in our modelling, with CAR typically at 25%. Volume differences in the formal sector as we move away from the centre are driven by height differences as assumed in the model. In contrast, in Table 3b column 3 and in Fig. 8, slum CAR declines as we move away from the centre, with the gradient slope of -0.090 used in calibration of the model in Table 1.

---

15 This is STATA local mean smoothing with an Epanechnikov kernel, with default settings.
16 There heights in the formal sector decline sharply as we move away from the city centre at a rate of 7.7% per km from column 2 of Table 1, while those in slums are flat or even rise slightly.
Again as assumed in the model and in contrast to the formal sector, slum volume differences across the city are driven by CAR differences.

In slums the cover to area ratio is very high, at 50-60% at 2-6 km out in the older slums nearer the city centre. The high CAR means that slums have little green/open space around houses and little in the way of real side streets. For the latter, we also give coverage including roads within each sector by dashed lines in the figure. Much more coverage by way of paved roads is added to the formal than the slum sector, where in the formal sector roads are about 15% of coverage near the centre.17

<table>
<thead>
<tr>
<th>Table 3. Height, CAR and BVAR gradients</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel a. Formal</strong></td>
</tr>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>Ln Formal Height</td>
</tr>
<tr>
<td>Distance to centre</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>R-squared</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Panel b. Slums outside of 2.5km</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>Ln Slum Height</td>
</tr>
<tr>
<td>Distance to centre</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>R-squared</td>
</tr>
</tbody>
</table>

**Volume [BVAR]:** Combining Figs. 7 and 8, slums produce housing with intense ground cover but little height while in the formal sector the opposite is the case. Fig. 9 gives this net, the built volume to area ratio (BVAR).18 Gradients for BVAR (column 4 in panels a and b of Table 3) for the formal sector and slums correspond respectively to those for formal height and slum CAR. In the formal sector up to almost 2 km from the centre, BVAR is very high, averaging around 7 cubic metres of space per metre of ground area. At 2km and beyond,

17 We know overall roads are about 22% of total area of the city centre, implying that roads in public sector use grid squares we have removed is high near the centre.

18 A graph on heterogeneity of BVAR by sector looks much like the one for height with lots of heterogeneity in the formal sector and much less heterogeneity in slums.
slums and the formal sector deliver essentially the same BVAR, so height and CAR differences cancel out. This is consistent with the theory where we saw that, dependent on parameters, cover in slums and height in formal areas could deliver similar housing volumes. At 6.5 km, the BVAR in slums does bump up; but, as we saw earlier in Fig. 5, that may be due to misclassification. For the opposing views of whether formal sector height trumps slum coverage in providing volume of built space or not, we have learned that, in Nairobi, they do equally well on average, albeit at very different quality levels.

**Total volume:** We now have the components of volume in eqn. (18) other than the share of land in slums vs. the formal sector. Fig. 10 pulls all the elements together. It shows total volume and then the share in the formal and slum sectors throughout the city. Given that BVAR from 2km and beyond is essentially the same in the formal and slum sectors, what drives the shares is land share. There are no slums within 2km of the centre, so the formal sector share of volume is 100%. Slums occupy no more than 20% of non-public land at any distance up to 10km from the centre and similarly slum share of volume never rises above 20%. There are two key takeaways. First slum volume is never a big part of the picture in part because it excludes formal sector commercial and industrial use. It is about 10% of total volume in 2015 and never exceeds 20% at any distance. Second total volume rises sharply to peak at almost 13.5 million cubic meters at 3.5 km from the centre as the amount of potentially available land in any circumference increases; but then it falls to average around 7-8 million. As we noted in Figs. 4-5, Nairobi has little available land beyond 4-5 km to the direct north and south.

### 3.4 Dynamics of Nairobi’s built environment

We now turn to changes in Nairobi’s built environment between 2003/4 and 2015, looking first at changes in built volume in total and by type, then at the different patterns of height and cover change by sectors and distance from the centre, and finally at building-by-building level demolition, redevelopment and infill. Our findings highlight the rapid evolution of developing country cities such as Nairobi.

**Urban built volume:** Volume changes occur at two margins, the intensive margin of the 2003/4 city area and the extensive margin defined as the difference between the 2003/4 and 2015 boundaries in Fig. 4. In the intensive margin, total built volume for non-public use increases by 48%, about a 3.6% annual increase. The extensive margin accounts for 17% of total volume in all uses in 2004, and 24% by 2015, amounting to a 120% increase in volume at that margin. Overall, within the 2015 boundary, total volume increases by 59%, about a 4.3% annual rate of increase. This compares with a similar annual population growth rate between the 1999 and 2009 censuses. The increase in formal sector volume of 60% is modestly more than the 55% for slums; and so slums only account for about 9.2% of the
increase in total volume. The small decline in the slum share of volume reflects slum population share stagnation: from the censuses in 1999 and 2009, the slum share of population in the 2015 city area declines modestly from 29.2 to 28.8%.

**Height and cover change by sector and distance:** Recall that eqn. (18) decomposes housing volume into elements, 
\[ v(x) = \frac{a(x)}{\text{total area}} \{ \rho_I \ BDVAR_I(x) + (1-\rho_I) BDVAR_F(x) \} \]. Over time we cannot distinguish the classification of cells by I or F, so therefore hold constant \( \rho_I \) and \( \rho_F \). What can change at any \( \chi \) (and hence \( x \)) are height and cover and hence \( BDVAR_I(x) = CAR_I(x) \cdot h_I(x) \). These then give the percent changes in \( v(x) \) and \( v_I(x) \).

We start with the spatial patterns of total volume change by sector. Given unchanged land shares, the BVAR percent changes are the same as the total volume percent changes. Fig. 11 shows volume changes by distance and the breakdown between the formal and slum sectors. Volume changes are large everywhere beyond 1-2 km from the centre. There are 45-70% increases in total volume from 2-8 km, highest at 3-5 km. Note that while there is dramatic change in the city, there is less change in the first km from the centre which is locked in by historical buildings and roads, and by sky-scrapers built over the last 35 years. The huge increases at 3-5 km out in already highly built areas are an aspect of focus. Until 9 km out within sector percent increases in the formal sector generally dominate those in the slums, showing the increasing relative role of the formal sector in the main part of the city. However, slum changes dominate at the city edge as the model predicts. In Fig. 11, since slums have such a small weight in total area, total changes generally mimic those in the formal sector.

Fig. 12 gives the height of redeveloped and unchanged buildings in the formal and informal sectors. In the formal sector, from 1.5 to 5 km, redeveloped buildings average twice the height of unchanged buildings. This is building taller with redevelopment, consistent with the model. In contrast in the slum sector, in Fig. 12, heights of unchanged and redeveloped buildings are almost the same, as assumed in the model.

Fig. 13 gives CAR changes by sector. For slums, the large increase in CAR mimics the volume changes, given mostly unchanged heights. Higher rents over time at all locations mean more intensive building, which given a fixed height mean more intensive use of land and crowding, as assumed in the model. For the formal sector, the CAR changes are largely in-fill (not captured in the model) as there is ongoing substitution away from ‘green’ space as a city grows. We explore this next.
In-fill, demolition, and churning: On the ground, changes are driven by a churning process of demolition and redevelopment, and by in-fill. Figs. 14 and 15 decompose the formal sector changes into these elements and indicate the high rate of churning that is taking place.

Fig. 14 gives the percentage changes in formal volume, in total and decomposed by infill, redevelopment and demolition. To highlight changes in the core of the city, we cut the x-axis at 8km (and give numbers at 10 km in the fn. to the figure). Demolition (without redevelopment) involves small coverage and volume changes throughout. Further from the centre where there is less initial development in slum or otherwise, infill starts to escalate. However, we can see that the increase in building volume nearer the city centre is dominated by redevelopment. The net increase in volume just at 3km out is over 35% due to redevelopment, with about another 20% from infill.19

To provide more detail, in Fig. 15 we turn to what we call churning of small lots. In solid lines we show the changes in formal sector building counts as a percent of 2003/4 counts and in dashed lines the same for area covered. Even within 1.4 to 4km about 35% of buildings are torn down and 50% of these are redeveloped. To benchmark the demolition rate of 35% we note in the USA from the American Housing Survey for 2009 to 2013, the annual rate of demolition and removal by disaster is 1.18% a year. For 11 years this would involve 12-13% of building removal. Nairobi is 3 times that, the outcome of a pace of redevelopment beyond the current USA experience.

While there is high degree of churning in counts, the coverage area involved is small. Demolition (without replacement) is 15-20% of 2003/4 building counts from 2-6 km out but only involves about 5% of cover. Infill by counts adds about 40% to 2003/4 counts from 1.5 to 4km out, but just 10-18% to 2003/04 cover. The 2003/4 footprint size involved in redevelopment from 2-6 km bounces along at 6-10% of 2003/4 cover, while the rate of counts redeveloped is double that. However, redeveloped buildings have a distinct and very large increase in average footprint size. The increase in footprint size of redeveloped buildings compared to the originals averages 100% at 3 km and rises to 200% by 6km.

In reality, building taller typically involves situations where coverage can be extended and/or land assembled to increase footprint size. In contrast infill and demolition involve historically smaller hemmed-in lots, on which it is hard to build high.20 Overall there is substitution away from green space as its opportunity cost rises, involving some conversion of small

19 The components do not add up exactly to the total because some buildings classified as unchanged exhibit small changes in drawing footprints between the years.
20 For example, in a sampling of 50 in-fill buildings from 0-1.5km, 32% involve building on top of small parking lots. Land released by demolition and still not redeveloped seems to go to other needed uses. In a sample of 50 demolitions from 0-1.5km, current usage is parking areas (27%), roads (15%), gardens for others (10%), and small sandwiched spaces (19%); only 29% are more open spaces, mostly with vegetation.
hemmed-in pieces of land to low height, small buildings. We believe hemmed-in usage reflects the lack of strong planning institutions and random timing of when different plots of land were able to be formalised.

4. Slum redevelopment, lack thereof, and welfare costs

4.1 Lack of slum redevelopment

To what extent is there slum redevelopment? We cannot accurately measure slum areas changes over time, although Figure 4 is suggestive; but we can look at changes over time within areas defined as slums in 2011. As we saw already, while there has been considerable redevelopment in these areas, the heights of unchanged and redeveloped slum buildings are similar. If one compares the distributions of redeveloped and unchanged heights of slum buildings in 2015 (not shown), the vast majority (70%) of both are 3m high. Further, there is little change at the upper end of the distribution. For example, 2.9% of unchanged slum buildings are over 9m high and only 3.4% of redeveloped slum buildings exceed 9m. These small changes are also clear from visual inspection of the high resolution satellite images we possess for 2004 and 2015.

4.2 Formalisation costs

The model suggests that persistence of slums relatively close to the centre is due to ‘formalisation costs’, a catch-all term for a complex array of barriers to land conversion. The literature suggests such barriers are particularly prevalent in government owned slums. In Nairobi, government ownership is 100% of slums near the centre, as indicated in Fig. 16, based on IPE 2012. Further from the centre the private share rises, although slum land is also held by the Nairobi City Council, mixed private and government, temporary occupation licenses, and road and riparian reserves.

The problems associated with slums on government owned land are discussed in a number of research studies and reports which point to the array of actors involved, to corruption, and to ‘outright plunder’ (Marx, Stoker and Suri 2013 and Southall 2005). Studies suggest slum housing is almost all rented and is operated by slum lords who make high profits. Guylani and Talukdar (2008) estimate payback periods on an investment in a single room of just 20.4 months. This is consistent with the fact that land is ‘free’ to slumlords; and, by our calibration, land’s share in revenue in the informal sector is 70%. In Kibera, of 120 slum lords surveyed, 41% were government officials, 16% (often the biggest holders) were politicians, and 42% were other absentee owners (Syagga, Mitullah, and Karirah-Gitau 2002 as cited in Gulyani and Talukdar 2008). The political economy issue is that if the government were to take the land and auction it for formal use, the slumlords would have no claim to the
revenue since they don’t own the land and their presence is at best quasi-legal. They would simply lose profitable businesses. Having well connected bureaucrats and political figures opposed to conversion presents a political problem.

Using Kibera as an example, the problem is accentuated by history. The 1000 acres in Kibera was awarded to Nubian soldiers in 1912. They occupied a portion of the land but at independence their claims (but not tenancy) were revoked, and land reverted to the government. The large portion of Kibera not occupied by Nubians was settled by others and had titles illegally allocated by local chiefs and bureaucrats. The moral claim of the Nubian descendants to at least the land they occupy is well recognized but the unwillingness to grant them title is yet another road block to redevelopment (Joireman and Vanderpoel, 2011).21

4.3 Welfare costs

The evidence indicates both persistence of slums relatively near the centre, and the presence of institutional and political obstacles to their formalisation. What are the welfare costs of this lack of formalisation? In our open city model welfare costs of inefficient land use are given by land rents foregone. To assess the cost of delayed formalisation we now use the model and estimated parameters to calculate the present value land rents earned by holding a piece of land in slum use for a given period of time, versus converting it to formal sector use.

We consider a piece of land at place $x$ and derive the present value of land rents earned from date $s$ onwards. The land is in informal use at date $s$ and is converted to formality at date $z$, $z \geq s$. We denote this present value $PV(x,s,z)$ and its value is

$$PV(x,s,z) = \int_s^z r_t(x,t)e^{-\rho(t-s)} dt + e^{-\rho(z-s)} \sum_{i=0}^{\infty} R_F(x,z+i\Delta t)e^{-i\Delta t}$$

$$= r_s(x,s) \frac{1-e^{-(\rho-\hat{\rho}\alpha/(\alpha-1))z}}{\rho-\hat{\rho}\alpha/(\alpha-1)} + r_F(x,s,s)\Phi \frac{e^{-(\rho-\hat{\rho}\gamma/(\gamma-1))(z-s)}}{1-e^{-(\rho-\hat{\rho}\gamma/(\gamma-1))\Delta t}} . \quad (19)$$

The first term on the left-hand side of (19) is the value in informal use, and integration uses eqn. (3) and the fact that revenue is growing at constant exponential rate $-(\rho-\hat{\rho}\alpha/(\alpha-1))$. The second term is value through repeated cycles of formal structures, and summation uses the relationship between the present value of land-rent and amortized flow rent at date of construction, $R_F(x,\tau_i) = \Phi(x,i)r_F(x,\tau_i,\tau_i)$, together with constant growth of revenue at rate

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21 Further documentation on the Nubian settlers in Kibera can be found online at Kenya’s Nubians, who also argue that the Nubians have a valid claim to the land in Kibera. (http://www.nubiansinkenya.com/)
\(- \hat{p}_F \gamma / (\gamma - 1)\) and constancy of \(\Phi\) across time and space. The present value rent loss from first formalising at date \(z\) rather than date \(s\) is \(PV(x, s, z) - PV(x, s, s)\).\(^{22}\)

To evaluate this expression, we need land rents. House rent data tell us revenue (i.e. rent of land and structure combined) per unit volume, and volume per unit land. Base numbers are in Table 4 and details are footnoted.\(^{23}\) Relating back to the theory, the gap in house rents reflects quality differences between sectors. There is a base quality difference in floor space provided in iron sheet or mud dwellings including facilities compared to permanent structures and then variable quality based on crowding (and implicitly lack of green space and side roads). Evaluating at \(x = 3.5\)km from the centre for \(s = 2015\), total house revenues are US$18.7 and US$66.9 p.a. on informal and formal land respectively, with BVAR and slum area from our building and mapping data. Fractions \((\alpha - 1)/\alpha = 0.7\) and \((\gamma - 1)/\gamma = 0.405\) of each accrue as land rent (using eqns. (4) and (9) and estimated values of \(\alpha\) and \(\gamma\)). Thus \(r_I(3.5,2015) = $13.09\) and \(r_F(3.5,2015,2015) = $27.1\).

<table>
<thead>
<tr>
<th>Table 4. Housing by sector at 3.5 km from city centre</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Formal</strong></td>
</tr>
<tr>
<td>House rent per month per sq m floor, 2012, Kenya shilling (KES)</td>
</tr>
<tr>
<td>BVAR 2015 (m³/m²)</td>
</tr>
<tr>
<td>Slum Land Area 2015 at 3-4 km (m²)</td>
</tr>
<tr>
<td>Annual rent per BVAR in 2015 USA dollars</td>
</tr>
<tr>
<td>Annual total revenue per unit land in 2015 USA dollars</td>
</tr>
<tr>
<td>Annual land rent per unit land in 2015 dollars</td>
</tr>
</tbody>
</table>

Calculating other terms in (19) requires knowledge of the rate of growth of prices and a discount rate. We assume (here and in the earlier calibrations) that \(\rho = 0.05\) and \(\hat{\rho} = 0.015\).

The latter is consistent with the current rate of price rise in local housing market of 8-9% with an inflation rate of about 6.5-7.5%.\(^{24}\) Solving the model earlier gave \(\Phi = 25.7\) and \(\Delta \tau = 65.6\). Routine calculation in (19) then yields the values in Table 5.

Comparison of the numbers in Table 5 tells us the present value (at date \(s\)) of land rent when formalisation takes place at different dates. Immediate conversion of land at 3.5km and from 2015 on yields present value land rent of $1217 per m², while perpetual delay yields $458 per

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\(^{22}\) Eqn. (19) corresponds to eqn. (10). Formalisation cost is a private but not social cost, so is absent from (19).

\(^{23}\) We take average monthly rent per sq. meter of floor space in 2012 by sector at \(x=3.5\)km in KES by 12 months, inflate by 8% a year for 3 years, (see fn. 24), multiply by the 2015 exchange rate of 0.01 and divide by 3 to get rent per cubic meter of space. We then multiply by BVAR by sector to get total revenue per unit land.

\(^{24}\) A Nairobi consulting firm reports real estate data. They calculate the annual rate of house rent increase for 2013, 2014 and 2015 ast 8.5%, 9.8%, and 9.6% respectively (Quarterly Reports at http://hasconsult.co.ke/).
Table 5. Present value of land rent: $ per m²

<table>
<thead>
<tr>
<th>Date of formalisation</th>
<th>PV to $∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>For ( z = 2015 )</td>
<td>( PV(3.5, 2015, 2015) = $1217 )</td>
</tr>
<tr>
<td>For ( z = ∞ )</td>
<td>( PV(3.5, 2015, ∞) = $458 )</td>
</tr>
<tr>
<td>For ( z = s + Δτ = 2081 )</td>
<td>( PV(3.5, 2015, 2081) = $909 )</td>
</tr>
<tr>
<td>For ( z = 2045 )</td>
<td>( PV(3.5, 2015, 2045) = $1092 )</td>
</tr>
</tbody>
</table>

The cost of perpetual informality is the difference, $759 per m², a loss of 62% of potential land value. There are 1.13 square kilometres of slum land in the distance band 3-4 km from the centre of Nairobi, of which we estimate about 10% is in roads and public schools (see Fig. 8), so about 1 square km is available for redevelopment. There is thus an aggregate gain from converting today over perpetual delay of about $759 million. For a perspective, suppose slumlords were compensated for conversion by $458 per m², as if they had the right to hang on forever. That leaves a remaining surplus of $759 per m². For the 25,000 households affected (occupying on average about 40 sq. meters of land), the gain is $13,000 per household. These households pay about $260 a year (median) in house rents.

Shorter delays are given in subsequent rows of Table 5. Over one building cycle, \( Δτ = 65.6 \), the loss from delaying conversion is $308 per m², (= $1217-$909). This can be expressed as a proportion of PV of land rents earned during the period of delay, and amounts to a loss of 44%. The cost of delay is lower for shorter periods as the last row of Table 5 indicates.

Gains from conversion decline with distance from the centre, via terms \( r_i(x,s) \) and \( r_p(x,s,s) \) in eqn. (19), where these vary in line with house rents according to \( e^{-θ(x/(a-1))} \) and \( e^{-θ(y/(y-1))} \) respectively. Thus, the gain from conversion per unit slum land in the distance band 4-5km, 1km further out from the centre, is modestly smaller. However, given the greater number of households affected (around 50,000) the aggregate involved is much larger.

5. Other Considerations

The model only examines residential housing capital and, in the data, we lump all built space together. We do not know building usage per se and in slums many buildings may have a dual residential-production purpose. We do have general land use maps. Much land at the centre is classified as in commercial use. Industry is in the eastern half of the city, with older industrial areas starting to the immediate south-east of the city centre and then stretching out.

25 If the delay is a full building cycle, then the PV of rents earned beyond \( s + Δτ = 2081 \), i.e. at the start of a building cycle, are the same regardless of earlier development and can be computed as $521 per m².
Other large industrial areas are to the north-west away from the centre. All these are far from Kibera in the western part of the city. We also can calculate the ratio of volume of built space to population in the formal and slum sectors. In the centre of the city, volume to population is very high given the intensity of commercial use. Volume to population in the formal sector falls with distance out to 2km; it then is similar to that in slums (which is flat throughout) until 4km, before rising again. Between 4 and 8 km, volume to population is high perhaps because of industrial uses.

Finally, we have ignored heterogeneity of the population as being beyond the scope of the paper. We note two things however. First overall higher socio-economic families tend to live nearer the centre. Second while slums have fewer household heads with college degrees (7% in slums versus 21% formal sector household heads) and more who terminate education after primary (standard 8, 23% in slums versus 13% in formal housing), the number graduating from high school is the same (24% of both of both slum and formal sector residents, 2009 Census). This indicates that slums are not populated overwhelmingly by very low education-income individuals.

6. Conclusions

This paper examined building development and redevelopment in a growing city and the welfare costs of institutionally created land market frictions. We modelled the dynamics of a growing city in which formal buildings are durable, but informal are not. We developed propositions about the timing and spacing of developments in the city. Building volumes decline with distance from the centre; but increase over time, by steps in the formal sector as redevelopment involves building taller. We took the model to a unique data set on the built environment of Nairobi for 2003/4 and 2015 and estimated key parameters to calibrate the model. We then formulated a measure of the welfare costs of institutional frictions in land markets, which plague many cities in the developing world.

For a large fast growing city like Nairobi, we found that in the core part of the city there is major redevelopment of 2004 formal sector buildings into taller new buildings, driving 50-60% increases in volume. There is high intensification of land use with infill of new buildings further from the city centre. While new slums tend to develop on the city fringes, we found that development of slum into formal sector housing mid-city over the 11 years is very slow.

Related to this slow conversion of slum to formal sector usage, we explored the role of formalisation costs. We argue that slum ‘ownership’ by the government in the core part of the city means unresolved land right issues and corruption with vested slum interests of political figures, creating artificially high formalisation costs. Even after paying off slumlords, conversion today would yield a surplus of about $13,000 per slum household in context.
where they pay about $260 a year for their housing. Poorly functioning land market institutions dramatically alter the built fabric of the city, creating a hotchpotch of building heights and uses and significant welfare losses. In the model we also explored the role of expectations in altering (re)development paths. Under-estimating future demand growth leads to stunted city heights and spatial size.

References


Figure 1: Urban development with perfect foresight

Figure 2a: Formalisation costs
Figure 2b: The hotchpotch: random variation in formalisation costs

![3D graph showing variation in building volume over distance from CBD and time.]

Figure 3: Spatial gradients; \( t = 140 \)

a: Building volume

![Graph showing ln(v) vs x for both formal and informal sectors.]

b: Land rent

![Graph showing ln(r) vs x for both formal and informal sectors.]

\[
\begin{align*}
\text{ln}(v_I) &\quad \text{ln}(v_F) \\
\text{ln}(r_I) &\quad \text{ln}(r_F)
\end{align*}
\]

\( x_0, x_1 \) mark the boundaries between formal and informal sectors.
Figure 4. City shape

Figure 5. 3-D average height of buildings by grid square in the formal and slum sectors
Figure 6. Vacant land prices per square meter land

Regression reported in Table 2 column 1.

Figure 7. Building height
a. Mean height in meters

Mean and percentiles based on pixel (3mx3m) heights.
Dashed lines add road cover to building cover in the numerator. Note road cover in the formal sector far exceeds that in slums. Roads defined as paved roads which can accommodate at least two cars passing.
Figure 12. Building height by development and sector

Figure 13. Changes in cover to area ratio (CAR)

Figure 14. Changes in volume: decomposition

At 10km the total volume increase is about 125%. Infill and net redevelopment approximate 120 and 20% each.

Figure 15. Changes in formal sector counts and cover due to infill, redevelopment and demolition, as proportion 2004.
Figure 16. Slum ownership
Appendix 1: Theory.

Derivation of equation (12) & (13): Derivation uses

\[
\frac{\partial R_i(x, \tau_i)}{\partial \tau_i} = -p_F(x,t)v_F(x, \tau_i) + \rho \int_{\tau_i}^{\tau_i^+} p_F(x,t)v_F(x, \tau_i^+)e^{-\rho(t-\tau_i^+)}dt
\]

\[
= -p_F(x,t)v_F(x, \tau_i) + \rho[R_F(x, \tau_i) + k_F(v_F(x, \tau_i))]
\]

and the fact that volume is optimised.

Parameters for figures: Parameter values in figures 1 – 3 are: \( \theta_F = \theta_I = 0.063, \gamma = 1.68, \alpha = 3.33 \). These gradients and elasticities come from Nairobi estimates as discussed in the text, and distance units are to be interpreted as km. The time dimension is based on setting \( \rho = 0.05 \), \( \hat{\rho}_I = \hat{\rho}_F = 0.015 \). Level variables are set at \( \bar{p}_F = 1, \bar{p}_I = 1, \kappa_F = 1/\alpha = 0.03 \), \( r_0 = 1 - 1/\alpha = 0.7, a_0 = 1 \). These set the numeraire (prices at \( t = 0 \)) and units such that informal development commences at \( t = 0 \) and edge informal development has unit volume per unit land. In the formal sector, \( \kappa_F = 10.5 \), this inducing formal sector development to take place at dates and times illustrated in Figs. 1 and 2.

Simulation is done with time running to \( t = 800 \), and reported up to \( t = 200 \). Distance runs to \( x = 50 \). In Fig 1 formalisation cost \( D = 0 \), and endogenous variables take values \( \Phi = 25.7 \) and \( \Delta \tau = 65.6 \). In Fig 2a, \( D \) is increased to 11 in distance band \( x \in [9, 14] \) and in 2b \( D \) takes positive standard normally distributed random variables times scale factor 10.

Closing the model:

Households: At date \( t \) a representative urban household living at distance \( x \) from the CBD receives income net of commuting costs \( w(t)T(x) \), where \( w(t) \) is the wage at date \( t \) (the same for all households), and \( T(x) \) is the fraction remaining after commuting costs. Each household has Cobb-Douglas preferences between housing and other goods,

\[
U(x,t) = \{a(x,t)s(x,t)w(t)T(x) - q(x,t)s(x,t)\}^{1-\beta}
\]

where \( s_i(x,t) \) is volume of housing \( q(x,t) \) is price, and \( a(x,t) \) is an amenity parameter. The corresponding indirect utility function is

\[
u(x,t) = \{a(x,t)/q(x,t)\}^\beta w(t)T(x)B, \quad B \equiv \beta^\beta (1 - \beta)^{1-\beta}.
\]

Free choice of location type means that, at any occupied location, utility takes a common city wide utility level, \( \bar{u} \), this giving bid-price

\[
q(x,t) = a(x,t)\{w(t)T(x)B/\bar{u}\}^{1/\beta}.
\]

There are two types of housing, formal and informal \( (i = F, I) \), and we allow preference parameters, commuting costs, amenity and prices to be type specific. Formal housing offers amenity equal to unity, and we write the bid-price as
\[ p_s(x,t) = \{w(t)T_s(x)B_s / \bar{u}\}^{\gamma / \beta_s} . \]

Informal housing offers amenity \( a(x,t) \), (depending on crowdedness, as in the text). We write the bid-price as
\[ q(x,t) = p_s(x,t)a(x,t) = a(x,t)\{w(t)T_s(x)B_s / \bar{u}\}^{\gamma / \beta_s} . \]

That is, we define \( p_s(x,t) = \{w(t)T_s(x)B_s / \bar{u}\}^{\gamma / \beta_s} \) and express the bid-price as the product, \( p_s(x,t)a(x,t) \), of amenity and an amenity free ('unit quality') price.

Constant exponential growth of the price of space is achieved by assuming that urban wages relative to outside utility (held constant at \( \bar{u} \)) grow at constant rate \( g \). Similarly, constant exponential decline with respect to distance is achieved by the share of income net of commuting declining with distance at rates \( \hat{T}_I, \hat{T}_F \), so \( p_i(x, t) = (\bar{w}e^{-\hat{T}_i x} / \bar{u})^{\gamma / \beta_i}, i = I, F. \)

This gives prices rising through time at constant rates \( \hat{p}_I = g / \beta_I, \hat{p}_F = g / \beta_F \), and declining with distance, \( \theta_I = -\hat{T}_I / \beta_I, \theta_F = -\hat{T}_F / \beta_F. \)

**Labour and population:** To complete the model, we note that population at a point is \( v/s \), total volume supplied divided by consumption of floor space per household. Total city population at date \( t \) is therefore
\[ L(t) = \sum_{i=1}^{\text{max}(t)} \int_{x_{i+1}(t)}^{x_i(t)} v_j(x, t) / s_p(x, t) dx + \int_{x_{i+1}(t)}^{x_0(t)} v_j(x, t) / s_f(x, t) dx . \]

The oldest formal development has been redeveloped the most times (which, at date \( t \), we denote \( \text{max}(t) \)). Notice that this expression assumes that the city is linear (or a set of rays), not a disc; modification to capture the latter is straightforward.

The final element is to close the model, either by setting \( \bar{u} \) exogenously with \( L(t) \) endogenous (open city), or with \( L(t) \) exogenous and determining the equilibrium citywide level of utility (closed city). The analysis in the body of the paper follows the open city route, with exogenous growth of urban wages relative to outside utility driving housing price growth.

**Appendix 2: Data Methodology**

This Appendix has two components. The first deals with measures on cover/footprint and volume we use to analysis. The second gives the algorithm used to extract unchanged buildings, redeveloped buildings and infill from the overlay of 2004 and 2015 depiction of building polygons.

**Measures of cover and volume**

Our unit of analysis is 150x150m grid squares. For calculating cover within the grid square in a usage, each of these is broken into 50 3x3m cells and use type classified by what is at the centroid of the 3m square in each period. There are three uses: vacant land, slum area and formal. For each 150x150 square we sum across the 50 cells to get total use of each type. Most 150x150 squares are either all slum or all formal sector. However there are about 12% which are mixed grid squares, for which we record the cover or volume of slum and formal separately.
Having summed the total area of use of each type in 3x3 squares in each 150x150 meter square, these are averaged for 150x150m squares whose centroid falls in a narrow distance ring. That sum is then divided by the total number of 150x150 grid squares in that distance band. For volume for 2015, for each 3x3m square which is formal sector, we have the height of the building at the centroid of that square. Volume for that 3x3 square is 9 times the height in meters of the building from LiDAR data. We then sum across the grids squares occupied with formal usage for 150x150m grid squares in each distance ring and then average by the total number of 150x150 m grid squares in the ring. For 2004 we have no height data. To infer 2004 heights, we use what we think is an upper bound on height: the height of unchanged buildings, where we presume demolished buildings between 2004 and 2015 are likely to be of lower height than those which survive. To assign a height to a 3x3m square in 2004 in formal sector usage, we take the average height in 2015 of all buildings that were there in 2004 for all 3x3m formal sector unchanged buildings in the own 150x150m grids square and its 8 queen neighbours. Height is the height assigned to each 3x3m square in usage in a distance ring from the centre averaged over all such cells, to effectively get a coverage weighted average of individual building heights.

How do we measure change between 2004 and 2015? For demolition, at the 3x3m level the square is defined as demolition if its centroid is covered by a 2004 building which has been replaced by open space. Demolished coverage is lost 2004 cover; demolished volume is assessed as before using the average height of unchanged buildings in the neighbourhood. Infill is new buildings which do now overlap with any 2004 buildings; a 3x3m square is infill if its centroid is covered by such a building on 2015 where there was no building in 2004. Infill cover and volume are assessed from 2015 data. Net redevelopment in coverage takes coverage in the new 2015 buildings and subtracts the coverage of old 2004 buildings. So for each 150m150m meter square we have for redeveloped buildings, we have total coverage in 2004 measured at the 3x3m level (centroid covered by the old 2004 building(s)) and we have total coverage in 2015 measured at the 3x3m squares (centroid covered by the new replacement 2015 building(s)). Net redevelopment at the 150x150square is the difference. In general, the same buildings are drawn in 2015 to have modestly more coverage than in 2004 so coverage change is likely to be an upper bound. Net volume change again assigns heights in 2004 to the 3x3m coverage based on neighbourhood averages for unchanged buildings and uses 2015 height information on the new buildings.

Overlaying Buildings

We match buildings across time by overlaying 2015 and 2004 building polygon data in order to track the persistency, demolition, construction and reconstruction of buildings over time. Since buildings are not identified across time our links rely on a shape matching algorithm. For each building, the algorithm determines whether it was there in the other period, or not, by comparing it with the buildings that overlap in the other time period.

This task is not straightforward, since the same building can be recorded in different ways depending on the aerial imagery used, whether building height was available, and the idiosyncrasies of the human digitizer.

Data and definitions

For 2004 we use a building dataset received from the Nairobi City Council with digitized polygons for every building, roughly 340,000 in the administrative boundary of Nairobi. For 2015 we use a similar dataset that was created by Ramani Geosystems using imagery (10-20cm resolution) and LiDAR (0.3-1m resolution). We have 2015 data for a wider extent, and consequently many more buildings, about 1.14 million. The LiDAR data in 2015 were used to measure heights of objects. With use of the aerial imagery and heights in 2015, a 3D model was created by hand, and rooftops extracted from this model.
The nomenclature we use is as follows. First, a trace is the collection of polygon vertices that make up its outline. A shape is the area enclosed by the trace, and can be thought of as a representation of the rooftop of a building. A cavity is an empty hole completely enclosed in a shape. A candidate pair is the set of any two shapes in different time periods which spatially intersect. A link is the relationship between a set of candidates in one period to a set of candidates in the other time period.

Pre-processing
Before running our shape matching algorithm we clean up the data sets. First we take care of no data areas. There are some areas that were not delineated in 2004, including the Moi Air Base, and Nairobi State House. We drop all buildings in these areas for both 2004 and 2015, amounting to roughly 1,500 buildings from the 2015 data, and 100 buildings from 2004. Next we deal with overlapping shapes, an issue arising in the 2015 data, although not that for 2004. This is most often the same building traced multiple times. We identify all such overlapping polygons and discard the smaller version until no overlaps remain; about 1,400 buildings from the 2015 data this way. We also drop small shapes, in part because the 2015 data has many very small shapes, while the 2004 data does not. In order to avoid complications of censoring in the 2004 data, we simply drop all shapes that have an area of less than 1m². We drop 2 small buildings in 2004, and 462 small buildings in 2015.

Another issue is that buildings are often defined as contiguous shapes in 2004, but broken up in 2015. For the majority of buildings we cannot aggregate the broken up pieces in 2015 since it is hard to identify such cases in general. To match these cases across time we rely on our one to many, and many to many matching algorithms defined below. However, in the specific case where a building is completely enclosed in another the task is much easier. First, we find all cavities present in each period, then we take all building shapes that overlap with the cavities in the same time period. After identifying all shapes that intersect a cavity, we redefine both shapes, the original shape containing the cavity and the shape intersecting it, as a single new shape.

Shape Matching Algorithm
After the pre-processing of each cross-section is complete, we run our shape matching algorithm to establish links between buildings across time periods. For any given building we consider 5 possible scenarios; that it has a link to no building, that it has a link to one building (one to one match), that it has a link to multiple buildings (one to many), that it is part of a group of buildings that match to one building (many to one), or that it is a part of a group of buildings that matches to a group of buildings (many to many). We follow and approach similar to Yeom et al (2015) however, due to the inherent difficulty of inconsistent tracings we contribute to their method by introducing the one to many and many to many approaches. We assign each link a measure of fit that we call the overlay ratio. We then choose optimal links based on the overlay ratio. Finally, we categorize links as matched or not using a strict cut-off on the overlay ratio of 0.5. Other cu-offs such as 0.4, 0.6 and 0.7 produced more errors in categorization.

Candidates
For all buildings A in the first time period, and B in the second time period we identify the set of candidates:

\[ CP = \{(A, B); \text{Area}(A \cap B) \neq 0\} \]

For each candidate pair we find the ratio of the intersection area over the area of each shape, so if shapes A and B intersect, we find \( r_{AB} = \frac{\text{Area}(A \cup B)}{\text{Area}(A)} \) and \( r_{BA} = \frac{\text{Area}(A \cap B)}{\text{Area}(B)} \).

We link all shapes which do not belong to a candidate pair to the empty set.
One to One Matching
First we consider candidate pairs to be links on their own. For each pair, we calculate the overlay ratio as the intersection area over union area, so if A and B are candidate pair, we find:

\[ R_{AB} = \frac{\text{Area}(A \cap B)}{\text{Area}(A \cup B)} = \frac{\text{Area}(A \cap B)}{\text{Area}(A) + \text{Area}(B) - \text{Area}(A \cap B)} \]

One to Many Matching
For each time period separately, we identify all candidate pair links for which their intersection to area ratio is above threshold \( \theta \). For shape A we define a group \( \mathcal{G} = \{ B; r_{BA} \geq \theta \} \). Now we calculate the overlay ratio of one to many links as the intersection area over union area ratio:

\[ R_{AG} = \frac{\text{Area}(A \cap \bigcup_{B \in \mathcal{G}} B)}{\text{Area}(A \cup \bigcup_{B \in \mathcal{G}} B)} = \frac{\sum_{B \in \mathcal{G}} \text{Area}(A \cap B)}{\sum_{B \in \mathcal{G}} \text{Area}(A \cup B)} \]

Many to Many Matching
Here we have two cases, one when the shapes are fairly similar, which we capture in previous sections (one to one, or many to one). The other is inconsistent shapes that form the same structure. To capture these we consider both time periods at once, we clean the candidate pair list, keeping links for which either ratio is above a threshold \( \theta_1 \):

\[ \mathcal{L} = \{(A, B); r_{AB} \geq \theta_1 \text{ or } r_{BA} \geq \theta_1\} \]

Then we condition to only keep shape for which the total ratio intersection is above threshold \( \theta_2 \), so shape A will be included if \( \sum_{B \in \mathcal{X} \cap \mathcal{Y} \in \mathcal{L}} r_{AB} \geq \theta_2 \). Now we are left with a new candidate list, which we convert to sets \( \mathcal{L}_C = \{(\mathcal{G}, \mathcal{H})\} \) and start merging them:

\[ \text{if } G_i \cap G_j \neq \emptyset \text{ or } H_i \cap H_j \neq \emptyset: \mathcal{L}_C = \{(G_i \cup G_j, H_i \cup H_j)\} \cup \mathcal{L}_C / \{(G_i, H_i), (G_j, H_j)\}, i \neq j \]

We keep doing this until we can no longer merge any two rows. At this point we calculate the overlay ratio of many to many links as the intersection area over union section ratio:

\[ R_{GH} = \frac{\text{Area}(\bigcup_{A \in \mathcal{G}} A \cap \bigcup_{B \in \mathcal{H}} B)}{\text{Area}(\bigcup_{A \in \mathcal{G}} A \cup \bigcup_{B \in \mathcal{H}} B)} \]

ICP Translation
We encounter a problem when the two shapes or groups of shapes are similar but do not overlap well, this usually stems from the angle at which the images were taken, and is especially prevalent with tall buildings. To address this issue, we translate one trace towards the other, and then recalculate the overlay ratio. As in Besl and McKay (1992), we use the iterative closest point (ICP) method to estimate this translation. To perform the ICP we ignore any cavity points as we found they often cause less suitable translation. We found that for similar shapes this will optimize the intersection area.

Optimal Linking
In the end, we rank all links by their overlay ratio. We iteratively keep the link with the highest overlay ratio, or discard it if at least one of the buildings in the link has already been confirmed in a separate link. From the list of optimal links, we define a link to be a match if its overlay ratio, or the overlay ratio after ICP translation is above 0.5. We then define all matched candidates as unchanged, and the remaining candidates as redeveloped. All buildings that were not considered as candidates are defined as infill, if from 2015, and demolished, if from 2004.

Accuracy Assessment
In order to assess the performance of the polygon matching algorithm we manually classified links between 2004 and 2015 for a random sample of buildings. We sampled 48 150x150m grid cells,
stratifying over slum, non-slum within 3km, non-slum within 6km, and non-slum further than 6km to the CBD. The sample consists of over 2,250 buildings in 2004 and 3,500 buildings in 2015.

Results

We first break down matches by their mapping type. There are five types of manual link: redeveloped/infill/demolished (0), one to one match (1), one to many match (2), many to one match (3), and many to many match (4). For the algorithm we further split (0) into infill/demolished (-1) and redeveloped (0). Appendix table 1 shows the correspondence between the two mappings by building (a) and roof area (b). We can see that most errors come from the one to one matches, however, the many to many matches have the worst performance. Overall the diagonal values are quite high, which means not only are we matching buildings well, but also the algorithm is recognising the clumping of buildings as a human does (bear in mind that, for example, the one to one matches which we ‘misclassify’ as many to many will still be classified as match in the final data). Finally, we have perfect correspondence for demolition and in 2015 nearly perfect for infill.

Next we compare buildings that were matched by the algorithm and those matched manually. For now we use a cut-off of the overlay ratio of 0.5, later we explore the effect of different cut-offs on performance. As seen in appendix table 1 infill and demolition are classified with almost perfect correspondence. For this reason we ignore buildings with these mappings and focus on accuracy of redevelopment and unchanged. In appendix table 2 we condense mappings 1, 2, 3, and 4 into category 1, while redevelopment, or category 0, remains the same.

We define precision $P$ (negative predictive value $NPV$) as the fraction of buildings classified as unchanged (redeveloped) by the algorithm that are correct, recall $R$ (true negative rate $TNR$) as the fraction of buildings classified as unchanged (redeveloped) by hand that the algorithm gets correct, and the F1 score ($F$) as the weighted average of the two.

$$P = \frac{True \ Positive}{Positive \ Predictions}, \quad NPV = \frac{True \ Negative}{Negative \ Predictions}, \quad R = \frac{True \ Positive}{Positive \ Condition},$$

$$TNR = \frac{True \ Negative}{Negative \ Condition}, \quad F = \frac{2 \times P \times R}{P + R}$$

The confusion matrix in table 2 is done across all sampled buildings in 2004 and weights observations by buildings (1) and roof area (2). The F1 score is high in both cases, but in part this is due to relative success classifying unchanged buildings: precision for buildings that were classified as unchanged (redeveloped) by hand that the algorithm gets correct, and the F1 score ($F$) as the weighted average of the two.

In our first attempt we arbitrarily picked 50% as a cut off of the overlay ratio. Here we take a closer look at this choice. Using our manually classified links we can maximize the F1 score with respect to the cut off. In appendix figure 1 we plot the F1 score weighted by roof area against cut-offs of the overlay ratio for the 2004 data. We find that the highest F1 score comes just below 50% suggesting our first estimate was not far off.

In figure 1 we plot lines for each method of calculating the overlay ratio: without ICP, with ICP, and the maximum of the two. Around 50% we can see that the maximum performs best, but with only a very slight improvement over the ICP alone, which is in turn marginally better than without the ICP.
Appendix Table 2.1 – Mapping Correspondence 2004

a) Weighted by Building

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<th>Algo=2</th>
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b) Weighted by Area (sq-m)

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</tr>
<tr>
<td>4</td>
<td>0</td>
<td>5317</td>
<td>5528</td>
<td>4795</td>
<td>4464</td>
<td>14262</td>
</tr>
</tbody>
</table>

Mapping definitions: -1 demolition or infill; 0 redevelopment; 1 one to one match; 2 one to many match; 3 many to one match; 4 many to many match

Appendix Table 2.2 – Matching all areas 2004

a) Weighted by Building

<table>
<thead>
<tr>
<th>Manual</th>
<th>Algo=0</th>
<th>Algo=1</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>433</td>
<td>88</td>
<td>0.83</td>
</tr>
<tr>
<td>1</td>
<td>137</td>
<td>1473</td>
<td>0.91</td>
</tr>
<tr>
<td>Precision</td>
<td>0.76</td>
<td>0.94</td>
<td>F=0.93</td>
</tr>
</tbody>
</table>

b) Weighted by Area (sq-m)

<table>
<thead>
<tr>
<th>Manual</th>
<th>Algo=0</th>
<th>Algo=1</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>28187</td>
<td>9679</td>
<td>0.74</td>
</tr>
<tr>
<td>1</td>
<td>10710</td>
<td>257888</td>
<td>0.96</td>
</tr>
<tr>
<td>Precision</td>
<td>0.72</td>
<td>0.96</td>
<td>F=0.96</td>
</tr>
</tbody>
</table>
Appendix Figure 2.1

![Graph showing F1 Scores under ICP Routines](image)

### Appendix: List of public uses

<table>
<thead>
<tr>
<th>Category</th>
<th>Public uses</th>
</tr>
</thead>
</table>
| Recreational                 | a) Impala club, Kenya Harlequins, and Rugby Union of East Africa (0.14kmsq)   
  b) Golf Course (0.9kmsq)   
  c) Arboretum (0.25kmsq)   
  d) Central park, Uhuru park, railway club, railway golf course (0.5kmsq)   
  e) Nyayo stadium (0.1kmsq)   
  f) City park, Simba Union, Premier Club (1.1kmsq)   
  g) Barclays, Stima, KCB, Ruaraka, Utali clubs, and FOX drive in cinema (0.3kmsq) |
| Undeveloped                  | a) Makdara Railway Yard (1kmsq)   
  b) John Michuki Memorial Park (0.1kmsq) |
| Special use -- Includes poorly traced areas | a) State House   
  b) Ministry of State for Defence   
  c) Forces Memorial Hospital and Administration Police Camp   
  d) Langata Army Barracks   
  e) Armed Forces   
  f) Moi Airbase |
| Educational (not primary and secondary schools) | a) University of Nairobi and other colleges   
  b) Kenya Institute of Highways & Built Technology   
  c) Railway Training Institute   
  d) Kenya Veterinary Vaccines Production Institute   
  e) Moi Forces Academy   
  f) NYS engineering, Kenya Institute of Monetary Studies, KCA university, KPLC training, Utali college |
| Public utility               | a) Dandora dump (0.5kmsq)   
  b) Sewage works (0.25kmsq)   
  g) Kahawa Garrison Public use   
  a) Communications Commission of Kenya (0.1kmsq)   
  b) Langata Womens prison (0.2kmsq)   
  c) Nairobi and Kenyatta hospitals, Milimani Police Station, Civil Service club   
  d) Mbagathi hospital, Kenya Medical Research Institute, Monalisa funeral home   
  c) National museums of Kenya   
  f) Kenya convention centre and railway museum   
  g) Industrial area prison   
  h) Mathari mental hospital, Mathare police station, traffic police, Kenya police, Ruaraka complex, and National youth service   
  i) Jamahuri show ground |